Prediction of ocular magnification and aniseikonia after cataract surgery

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ABSTRACT.

Background: Ocular magnification and aniseikonia after cataract surgery has been widely ignored in modern cataract surgery. The purpose of this study was to analyse ocular magnification and inter-individual differences in a normal cataract population with a focus on monovision.

Methods: From a large dataset containing biometric measurements (IOLMaster 700) of both eyes of 9734 patients prior to cataract surgery, eyes were indexed randomly as primary (P) and secondary (S). Intraocular lens power (IOLP) was derived for the HofferQ, Haigis and Castrop formulae for emmetropia for P and emmetropia or myopia (-0.5 to -2 dpt) for S to simulate monovision. Based on the pseudophakic eye model in addition to these formulae, ocular magnification was extracted using matrix algebra (refraction and translation matrices and a system matrix describing the optical property of the entire spectacle corrected or uncorrected eye).

Results: With emmetropia for P and S the IOLP differences (S-P) showed a standard deviation of 0.162/0.156/0.157 dpt and ocular magnification differences yielded a standard deviation of 0.0414/0.0405/0.0408 mm/mrad for the HofferQ/Haigis/Castrop setting. Simulating monovision, the myopic eye (S) showed a systematically smaller mean absolute spectacle corrected ocular magnification than the emmetropic eye ($-0.0351/-0.0340/-0.0336$, respectively, relative magnification around 2%). If myopia in the S eye remains uncorrected, the reduction of ocular magnification is much smaller (around 0.2–0.3%).

Conclusion: Vergence formulae for IOLP calculation sometimes implicitly define a pseudophakic eye model which can be directly used to predict ocular magnification after cataract surgery. Despite a strong similarity of both eyes, ocular magnification does not fully match between eyes and the prediction of ocular magnification and aniseikonia might be relevant to avoid eikonic problems in the pseudophakic eye.

Key words: aniseikonia – biometry – interocular symmetry – anisometropia – paraxial optics – lens power calculation – matrix calculation – ocular magnification

Background

In modern cataract surgery aniseikonia, defined as a disparity between the retinal image size of the left and the right eye of an individual is mostly ignored (Achiron et al. 1997; Rutstein et al. 2006; Langenbucher & Szentmáry 2008). Researchers and clinicians focus mainly on image performance, new lens designs promising reduced optical aberrations, or on recovery of pseudophakic pseudo-accommodation using multifocal optics, enhanced depth of focus implants or monovision.

However, one mostly overlooked reason for patient dissatisfaction is postoperative disparity of retinal image sizes in both eyes of an individual. This can lead to headache, fusion problems, or in severe cases, to a loss of stereopsis (Langenbucher & Szentmáry 2008; Rajan et al. 2008; Krarup et al. 2021). From the literature, we know that in untreated eyes aniseikonia is mostly in a range of ½% (Krarup et al. 2021). An image size disparity of up to 2 or 3 percent is well tolerated by most patients, but retinal image size differences of 3% or more are sufficient to cause rapid fatigue. Therefore, image size disparity is of high clinical relevance, since such a mismatch in image size can result in patient dissatisfaction after cataract surgery even if target refraction is perfectly achieved. This implies that some patient complaints

can be explained directly in terms of aniseikonia. The main reason for this not being considered in daily clinical routine is that direct measurement of ocular magnification or aniseikonia is difficult and sometimes unreliable. There are existing computer-based test strategies for aniseikona (Rutstein et al. 2006; Fullard et al. 2007) or classical test strategies (Willeford et al. 2020; Krarup et al. 2021).

In most cases, aniseikonia after cataract surgery results from a disparity of biometric measures (anisometropia) such as axial length, anterior segment dimensions or corneal curvature (Rajan et al. 2008; Hashemi et al. 2013; Li & Bao 2014; Kansal et al. 2018; Krarup et al. 2021). However, we also have to be aware that enhanced monovision for recovery of pseudo-accommodation could be one of the major reasons for aniseikonia, and this is mostly ignored in the daily routine.

During ocular biometry (obligatory prior to cataract surgery), all relevant data required for prediction of the ocular magnification of both eyes are available (Fişuş et al. 2021). Ocular magnification is determined by the power of all refractive surfaces as well as the interspaces between the surfaces (each with their characteristic refractive indices) between the object and the retinal image (Langenbucher et al. 2003). This means that the predicted postoperative spectacle refraction (target refraction, TR), corneal front and back surface power, the power of the intraocular lens implant (IOL), the back vertex distance of the spectacle, the central corneal thickness, the aqueous depth, the central thickness of the lens implant, the vitreous depth, as well as the refractive indices of the aqueous humour, the lens implant and the vitreous humour, all affect the overall object to image magnification. In a pseudophakic optical model, the intraocular lens implant may be simplified as a 'thin lens' characterized by its refractive power, and in cases where the corneal thickness or corneal back surface curvature cannot be directly measured, the cornea may also be simplified as a thin lens (Haigis 2009). The ocular magnification can be derived from raytracing (Langenbucher et al. 2021b), but for a prediction of postoperative aniseikonia a calculation concept using linear Gaussian optics (restricted to the paraxial space) has been shown to be sufficient (Langenbucher et al. 2005).

There are several options for matching the ocular magnification between eyes, especially in cataract surgery with intraocular lens implantation: the ocular magnification can be modulated using a combination of intraocular lens power and a spectacle or contact lens refraction (target refraction), or optionally using special eikonic intraocular lens designs, which modulate ocular magnification by means of the ratio of front to back surface curvature (with the shape factor of the lens) (Achiron et al. 1997; Langenbucher et al. 2005. This is restricted to small changes resulting from the small central thickness of the lens).

The purpose of the present study was

- to develop and present a concept for predicting the ocular magnification of both eyes in the postcataract situation based on ocular biometry and linear Gaussian optics,
- to predict aniseikonia as the ocular magnification disparity in a large cataract population for use as normative data, if the lens power is calculated for postoperative emmetropia,
- to define a linear multivariable prediction model for ocular magnification which, in addition to biometric data, includes a target refraction (at the spectacle plane, if the lens power is calculated for ametropia), and
- to investigate the effect of (uncorrected and spectacle corrected) monovision on the ocular magnification.

Methods

Dataset for our analysis

We used a dataset containing in total 32 198 biometrical measurements from the IOLMaster 700 (Carl-Zeiss-Meditec, Jena, Germany) from two clinical centres (Augenklinik Castrop, Castrop-Rauxel, Germany and Department of Ophthalmology, Johannes Kepler University Linz, Austria) for this retrospective study. All measurements were performed in a cataractous population, excluding pseudophakic eyes. Duplicate measurements of eyes, eyes in pharmacologically stimulated mydriasis (pupil width more than 5.2 mm), and incomplete records in the dataset were discarded. Measurement data indexed as being after refractive surgery, or having ectatic corneal diseases (keratoconus, keratoglobus, pellucide marginal degeneration) or other corneal pathologies were also omitted from the dataset. The data were transferred to a .csv data table using the data export module of the IOLMaster 700 software. Data tables were reduced to the relevant parameters required for our data analysis, consisting of: laterality (left or right eye), patient's date of birth and examination date of the eyes, curvature of the corneal front surface in the flat (R1) and the steep (R2) meridian both in mm, axial length of the eye (AL in mm), central corneal thickness (CCT in mm), anterior chamber depth (ACD) measured from the corneal front apex to the crystalline lens front apex in mm, central thickness of the crystalline lens (LT in mm). The data were transferred to Matlab (Matlab version 2019b, MathWorks, Natick, USA) for further processing. A waiver was provided for this study by the local ethics committee (Ärztekammer des Saarlandes, 157/21).

Preprocessing of the data

Custom software for data processing and analysis was written in Matlab. From the entire dataset, we selected patients with bilateral measurements taken on the same examination day, with all other examinations being discarded. Each patient's age (Age in years) was derived from their date of birth and the examination date. From the curvature of the corneal front surface (R1, R2), we derived the average curvature ($R12 = 0.5 (R1 + R2)$), and the mean corneal power using the Javal keratometer index ($K12 = 0.5 (337.5/R1 + 337.5/R2)$).

The dataset was split into primary eyes (P) and secondary eyes (S) using a pseudo-random sequence indexing one eye of each individual as P and the other eye was indexed as S. The differences between the biometric measures of P and S eyes were recorded. In a first scenario, to analyse the differences in predicted IOL power and predicted ocular magnification, the TR for all eyes was set to zero. In a second scenario, to investigate the effect of monovision on ocular magnification,

the TR for IOL power calculation for the P eyes was set to zero and for the S eyes was set to a uniformly distributed random value between −0.5 and −2.0 dpt. In this context, the ocular magnification was derived for the P eyes (with TR equals zero), and the spectacle corrected and the uncorrected ocular magnification (with a myopic TR) were extracted for the S eyes (Langenbucher et al. 2003). For a third scenario, the TR was set to a uniformly distributed random value between 0.0 dpt and −2.0 dpt for all eyes (P and S), in order to define a linear prediction model for the ocular magnification from the biometric measures and the TR.

Intraocular lens power calculation and prediction of ocular magnification

Three different vergence based calculation formulae were used for calculating the intraocular lens power: The HofferQ formula (Hoffer 1980; Hoffer 1993), the Haigis formula (Haigis et al. 2000) and the Castrop formula (Wendelstein et al. 2021; Langenbucher et al. 2021a). The HofferQ formula and the Haigis formula are based on a pseudophakic schematic model eye with 3 refracting surfaces (TR at spectacle plane, cornea as thin lens and IOL as thin lens). In the HofferQ formula, the IOL power is calculated from AL, K12 derived using the Javal keratometer and TR. In the Haigis formula, the IOL power is determined from AL, R12 and TR. In contrast to the HofferQ and Haigis formulae, the Castrop formula uses a pseudophakic schematic model eye with four refractive surfaces (TR at spectacle plane, cornea as a thick lens with front and back surface and IOL as a thin lens). Without loss of generality, the formula constants were extracted from the IOLCon WEB site (https://iolcon.org, accessed on 02.02.2022) for the Hoya Vivinex lens (Hoya Surgical Optics, Singapore). From the respective IOL power calculation concepts, we extracted the refractive power of the cornea (either the power of the thin cornea for the HofferQ and the Haigis formula or the power of the thick cornea for the Castrop formula), the refractive indices for aqueous humour and vitreous, and the predicted effective lens position (ELP) of the thin IOL. Using these model parameters,

the ocular magnification was derived using matrix algebra (Haigis et al. 2000; Langenbucher et al. 2003): the refractive surfaces were characterized using refraction matrices, and the interspaces, each modelled as a homogeneous optical medium, were characterized using translation matrices. From the 2 × 2 refraction and 2 × 2 translation matrices, we calculated a system matrix, which provides the equivalent power of the eye (upper right element) as well as the ocular magnification (lower left element). The calculation scheme for determining the ocular magnification using matrix algebra has previously been shown in detail in the literature (Langenbucher et al. 2003).

In the case of a corrected eye (TR equal to zero or TR nonzero with a corresponding spectacle correction), all rays passing the refracting surfaces on the optical path from the object to the image plane intersect the image plane at the focus. In contrast, in the case of an uncorrected eye (TR nonzero without spectacle correction) we obtain a blurry image at the focal plane, and the chief ray passing through the pupil centre must be identified (Langenbucher et al. 2003; Langenbucher et al. 2021b). This pupil centre is assumed to be located on the optical axis at a distance ACD behind the corneal apex. This procedure of isolating the chief ray is important for scenarios 2 (for S) and 3 (P and S) in order to extract the ocular magnification for the uncorrected eye (Haigis et al. 2000; Langenbucher et al. 2007).

Statistics and linear prediction model for ocular magnification

The biometric data and the respective differences between S and P eyes are shown descriptively with mean (MEAN), standard deviation (SD), median (MEDIAN) as well as the lower and upper boundaries of the 90% (CL90L and CL90U) and 99% (CL99L and CL99U) confidence intervals. In an explorative analysis, the ocular magnification is shown for scenarios 1 and 2 (and also the uncorrected ocular magnification for S eyes in scenario 2).

The relevant parameters determining the ocular magnification were identified using a stepwise linear regression algorithm (Draper & Smith 1998). Based

on a significance level of 0.05, parameters are added to the initial regression model (constant model) and those having a statistical significance of 0.05 or higher excluded. The number of iterations was restricted to 100, and a step size of less than or equal to 1e-12 was used to terminate the iteration. A linear prediction model for prediction of ocular magnification was then defined for all 3 IOL power calculation concepts from the relevant parameters in terms of minimizing the root mean squared model prediction error.

Results

After quality approval of the dataset and filtering out incomplete data and patients with only one eye measured, a total of N = 19 468 measurements of (9734 right and 9734 left eyes from 9734 patients) were enrolled in our study. From the entire dataset, 9734 eyes (4857 left and 4857 right eyes) were indexed as P and 9734 eyes (4857 left and 4857 right eyes) as S.

The mean age of the study population was 69 ± 15 years (median 73 years, 90% confidence interval from 43 to 85 years). The mean corneal curvature R12 of the corneal front surface was 7.71 ± 0.27 mm, median 7.71 mm, 90% confidence interval 7.29 mm to 8.16 mm. For use in the Castrop formula, the mean corneal back surface was calculated from the corneal front surface curvature (separately for both principal meridians) using a factor of 0.84. Table 1 shows the explorative data for the biometric parameters AL, CCT, ACD, LT, R12 and K12 for the entire dataset (N = 19 468 eyes) and the difference S – P (values shown are scaled by ×7100). The SD of the intra-individual difference is 0.3745 mm/11.2 μm/0.1347 mm/0.2051 mm/0.0814 mm/0.4772 dpt for AL/CCT/ACD/LT/R12/K12.

Figure 1 displays the cumulative density function (CDF) graph of the absolute value of the intra-individual differences between both eyes (S – P) for the biometric parameters AL, CCT, ACD, LT, R12 and K12. The AL shows the largest differences between both eyes (mean absolute difference 0.2122 mm), whereas the CCT shows the smallest differences (mean absolute difference 6.9 μm).

In Table 2, the descriptive data for the predicted IOL power for

Table 1. Explorative data of ocular biometry in the cataract population.

		AL in mm	CCT in mm	ACD in mm	LT in mm	R12 in mm	K12 in dpt
All	Mean	23.6769	0.5521	3.1324	4.6130	7.7123	43.8227
N = 19	SD	1.4015	0.0368	0.4171	0.4903	0.2704	1.5373
468	Median	23.4866	0.5513	3.1257	4.6417	7.7075	43.7959
	CL90L/CL90U	21.8407/26.1416	0.4932/0.6137	2.4483/3.8437	3.6698/5.3409	7.2849/8.1625	41.3510/46.3380
	CL99L/CL99U	20.8160/29.7897	0.4559/0.6520	2.1194/4.2091	3.2116/5.7745	7.0285/8.4662	39.8674/48.0344
S − P	Mean	−0.2532	−0.0020	0.0093	−0.1201	0.0514	−0.3584
(×100)	SD	37.4462	1.1184	13.4467	20.5132	8.1410	47.7242
N = 9734	Median	0.0722	0.0060	−0.0024	−0.0860	0.0870	−0.3893
	CL90L/CL90U	−44.9958/ 42.2795	−1.4173/ 1.4040	−20.5071/ 20.9969	−31.3441/ 29.9309	−11.3652/ 11.5138	−65-3139/64.4921
	CL99L/CL99U	−196.5458/ 176.8569	−2.9855/ 3.0395	−45.4627/ 46.5480	−68.1349/ 67.6806	−24.7454/ 25.0722	−147.3529/ 148.6900

The upper section refers to biometry of 19 468 eyes of 9734 patients. The lower section shows the difference in biometric data between the secondary (S) and primary (P) eye (after splitting randomly; please note that all values in this section are scaled by ×100). AL, CCT, ACD, LT, R12, K12 refer to axial length, central corneal thickness, anterior chamber depth, lens thickness, mean radius of the corneal front surface, and mean corneal power derived from corneal front surface radius using the Javal keratometer index. Mean, SD, Median, CL90L/CL90U, CL99L/CL99U refer to the mean, standard deviation, median, and lower/upper boundary of the 90% and 99% confidence interval.
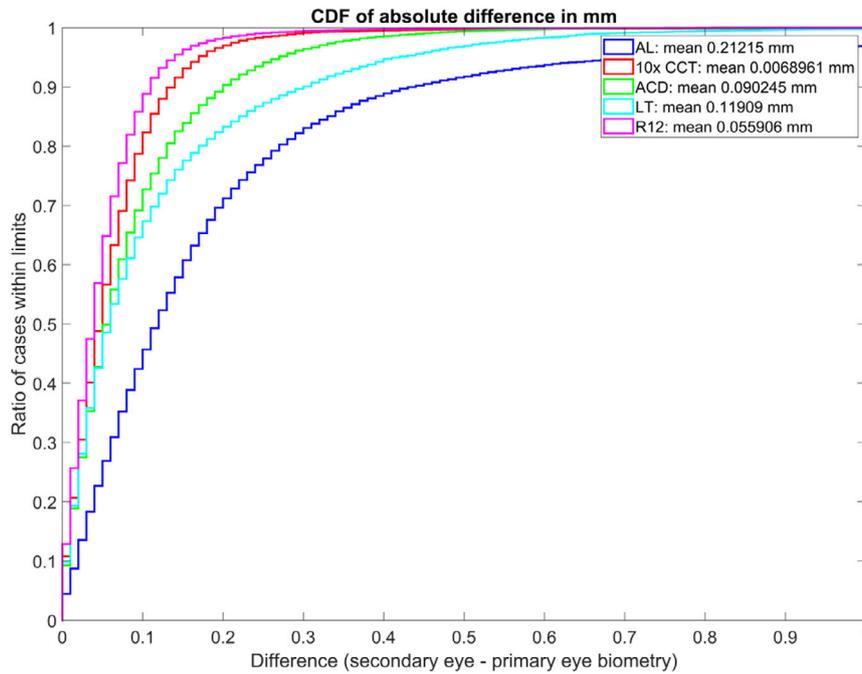

Fig. 1. Cumulative density function (CDF) plot of the absolute value of the difference between the secondary (S) and the primary (P) eye (S-P) for axial length (AL), central corneal thickness (CCT, scaled by ×10), anterior chamber depth (ACD), lens thickness (LT) and mean corneal front surface radius (R12). The intra-individual differences (between both eyes of an individual) in AL (blue line) seem to be much larger compared with the differences in ACD or LT. The smallest difference is observed in CCT.

emmetropia (scenario 1) calculated using the HofferQ (formula constant pACD = 5.7356), the Haigis (formula constants a0/a1/a2 = −0.6853/0.3417/0.2029) and the Castrop formula (C/H/R = 0.3420/0.1430/0.0192) are shown together with the predicted postoperative ocular magnification for the primary and secondary eyes. The difference between the predicted IOL power and the ocular magnification is shown in the lower section of this table. The SD of the intra-individual

difference is 1.6231 dpt/4.14 e-4 mm/mrad with the HofferQ formula, 1.5552/4.05 mm e-4 mrad/mm for the Haigis formula and 1.5726 dpt/4.08 e-4 mm/mrad for the Castrop formula.

Fig. 2 shows the cumulative density function (CDF) plot of the absolute value of the difference between the secondary (S) and the primary (P) eye (S-P) for the predicted intraocular lens power (for emmetropia, scenario 1) using the HofferQ, Haigis and Castrop formula (upper graph), and the absolute

value of the respective predicted postoperative difference in the predicted ocular magnification based on the models underlying the formulae under test (lower graph). The mean absolute difference in the IOL power is 0.8632 dpt/0.8377 dpt/0.8526 dpt for the HofferQ/Haigis/Castrop formula; the mean absolute difference in the ocular magnification based on the eye models used for the HofferQ/Haigis/Castrop formula are 1.91 e-4 mm/mrad/1.87 e-4 mm/mrad/1.91 e-4 mm/mrad.

Table 2. Descriptive data of the predicted intraocular lens power (IOLP, for the Hoya Vivinex lens as an example) using the HofferQ, Haigis and Castrop formulae and the respective predicted ocular magnification (OM) after cataract surgery.

IOL formula		HofferQ formula		Haigis formula		Castrop formula	
		IOLP in dpt	OM	IOLP in dpt	OM	IOLP in dpt	OM
P	Mean	20.7684	0.0165	20.7928	0.0166	20.6802	0.0167
N = 9734	SD	4.5088	0.0012	4.2421	0.0012	4.3193	0.0012
	Median	21.4358	0.0164	21.4391	0.0164	21.3204	0.0165
	CL90L/ CL90U	12.3892/26.6161	0.0151/0.0186	12.8780/26.2377	0.0152/0.0187	12.5897/26.3115	0.0152/0.0188
	CL99L/ CL99U	1.8996/31.6945	0.0142/0.0219	3.1053/31.3022	0.0144/0.0218	3.1969/31.6117	0.0144/0.0219
S	Mean	20.7801	0.0165	20.8033	0.0166	20.6896	0.0167
N = 9734	SD	4.4724	0.0012	4.2067	0.0012	4.2826	0.0012
	Median	21.4294	0.0164	21.4423	0.0164	21.3206	0.0165
	CL90L/ CL90U	12.6778/26.5599	0.0151/0.0186	13.1112/26.2033	0.0152/0.0186	12.8215/26.2355	0.0152/0.0188
	CL99L/ CL99U	2.1813/31.5465	0.0142/0.0220	3.3480/31.2328	0.0144/0.0220	303 003/31.6435	0.0144/0.0221
S − P	Mean	1.1621	−0.0002	1.0517	−0.0002	0.9338	−0.0002
(×100)	SD	162.3059	0.0414	155.5192	0.0405	157.2558	0.0408
N = 9734	Median	0.3773	0.0000	0.8706	0.0001	0.0772	0.0001
	CL90L/ CL90U	−178.2315/ 182.1621	−0.0384/ 0.0382	−173.6345/ 175.0122	−0.0376/ 0.0376	−177.2748/ 178.1828	−0.0387/ 0.0382
	CL99L/ CL99U	−666.0883/ 710.4430	−0.1772/ 0.1609	−638.4250/ 671.0177	−0.1738/ 0.1558	−644.9981/ 681.9468	−0.1732/ 0.1597

The first and second sections refer to the data after splitting the 19,468 eyes randomly into primary (P) and secondary eyes (S), with each patient contributing one eye to the P and one eye to the S group. The last section shows the difference in biometric data between the secondary (S) and primary (P) eye (please note that all values in this section are scaled by ×100). Mean, SD, Median, CL90L/CL90U, CL99L/CL99U refer to the mean, standard deviation, median, and lower/upper boundary of the 90% and 99% confidence interval.

Table 3 summarizes the explorative data for scenario 2 (simulated monovision with the primary eye calculated for emmetropia and the secondary eye calculated for a target refraction randomly distributed between −0.5 and −2.0 dpt). In this table, the difference between the secondary and the primary eye (S − P) is shown for TR, for the predicted IOL power calculated with the HofferQ, Haigis and Castrop formula, and for the predicted ocular magnification. For the P eye targeting for emmetropia, there is no difference between the (spectacle) corrected and uncorrected ocular magnification. In contrast, for the S eye the corrected ocular magnification refers to the magnification derived with an adequate spectacle correction, whereas the uncorrected ocular magnification refers to the magnification derived from the chief ray of the blurry image. The mean corrected ocular magnification in the S eyes is systematically lower than in the P eyes (−3.51 e-4 mm/mrad for the HofferQ, −3.40 e—4 mm/mrad for the Haigis, and −3.36 mm/mrad for the Castrop formula). In contrast, the uncorrected ocular magnification (for the blurry image) in S eyes is only slightly smaller than in P eyes (5.7 e-5 mm/mrad for HofferQ, −4.3 e-5 mm/mrad for Haigis and −3.8 e-5 mm/mrad for Castrop).

Figure 3 shows the combined scatterplot and histogram of the difference in corrected ocular magnification (S–P) versus the mean of the ocular magnification (S/P) for scenario 2. The calculation was performed using 3 pseudophakic models, according to the HofferQ, Haigis and Castrop formulae. The histogram on the left shows that, as a result of the myopic refraction in S eyes, the ocular magnification in S is systematically smaller compared with P eyes. From the histogram shown below the scatterplot, it can be seen that the Hoffer Q formula (blue line) shows a slight shift to the left, indicating that the ocular magnification prediction is slightly lower, whereas the Haigis and the Castrop formulae show slightly larger values for the ocular magnification.

Finally, linear multivariable prediction models were derived to estimate ocular magnification from the biometric data and the target refraction used in the IOL calculation scheme. In scenario 3, the relevant parameters for the linear prediction model of (corrected) ocular magnification were identified for the pseudophakic model of the HofferQ (Model 1), Haigis (Model 2) and Castrop formula (Model 3) using a stepwise linear regression model. For all 3 models, the input parameters AL, CCT, ACD, LT and R12/K12 have a significant effect on the ocular magnification. To be in accordance with the formula layout, we restricted our input parameters to TR, AL and K12 for Model 1, TR, AL, ACD and R12 for Model 2, and TR, AL, CCT, ACD, LT and R12 for Model 3.

In Model 1, ocular magnification OM reads

$$OM = -7.111 \cdot 10^{-3} + 8.545 \cdot 10^{-4}$$
$$\cdot AL + 7.809 \cdot 10^{-5} \cdot K12 + 2.783$$
$$\cdot 10^{-4} \cdot TR$$

The mean squared fit error was 4.78 e-5 mm/mrad and the coefficient of determination R^2 was 0.998. All input parameters as well as the intercept showed a significance value P < e-100.

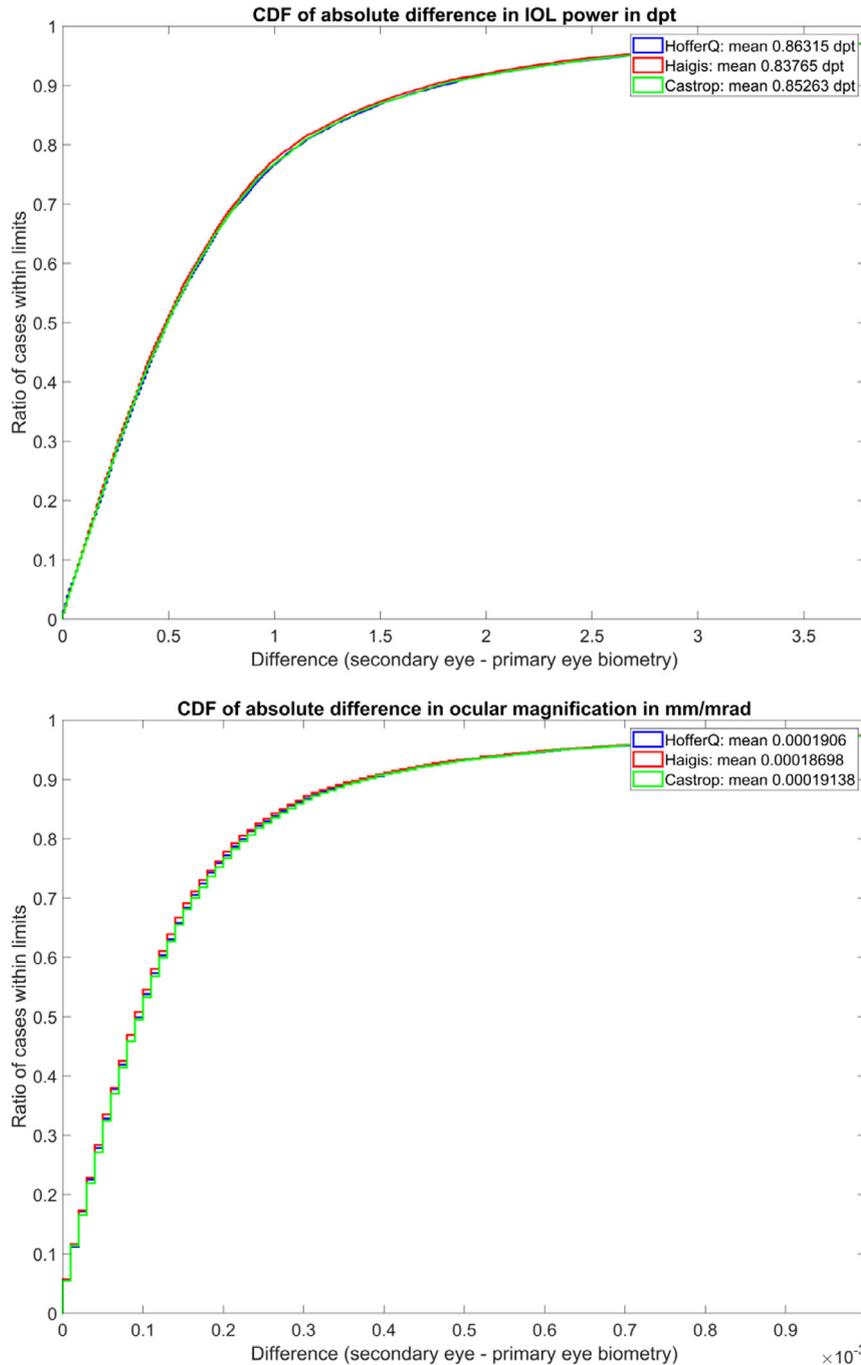In Model 2, the ocular magnification OM reads

Fig. 2. Cumulative density function (CDF) plot of the absolute value of the difference between the secondary (S) and the primary (P) eye (S-P) for the predicted intraocular lens power using the HofferQ, Haigis and Castrop formula (upper graph) and the respective predicted postoperative absolute value of the difference for the predicted ocular magnification based in the pseudophakic eye model of the HofferQ, Haigis and Castrop formulae (lower graph). There is no systematic difference in the CDF between the 3 formulae. The mean absolute difference is shown in the legends of the graphs.

$$OM = 4.2871 \cdot 10^{-4} + 8.522 \cdot 10^{-4}$$
$$\cdot AL - 1.179 \cdot 10^{-4} \cdot ACD - 4.685$$
$$\cdot 10^{-4} \cdot R12 + 2.693 \cdot 10^{-4} \cdot TR$$

The mean squared fit error was 3.60 e-5 mm/mrad and the coefficient of determination R^2 was 0.999. All input parameters as well as the intercept showed a significance value P < e-100.

In Model 3, ocular magnification OM reads

$$OM = 2.469 \cdot 10^{-4} + 8.740 \cdot 10^{-4} \cdot AL$$
$$+ 7.680 \cdot 10^{-5} \cdot CCT - 2.127$$
$$\cdot 10^{-4} \cdot ACD - 7.834 \cdot 10^{-5} LT$$
$$- 4.224 \cdot 10^{-5} R12 + 2.660$$
$$\cdot 10^{-4} \cdot TR$$

The mean squared fit error was 2.23 e-5 mm/mrad and the coefficient of determination R^2 was 0.999. Central corneal thickness (CCT) shoed a significance of 1.56 e-66, all other input parameters as well as the intercept showed a significance value P < e-100.

Figure 4 displays a combined scatterplot and histogram of the performance for Models 1, 2 and 3 according

Table 3. Descriptive data of the difference between secondary and primary eye (S-P) in predicted intraocular lens power (IOLP, for the Hoya Vivinex lens as an example) using the HofferQ, Haigis and Castrop formulae and in predicted ocular magnification (OM, values scaled ×100) after cataract surgery.

S – P, N = 9734	TR	HofferQ formula			Haigis formula			Castrop formula		
		IOLP	Corrected OM (×100)	Uncorrected OM (×100)	IOLP	Corrected OM (×100)	Uncorrected OM (×100)	IOLP	Corrected OM (×100)	Uncorrected OM (×100)
Mean	−1.2456	1.8630	−0.0351	−0.0057	1.7659	−0.0340	−0.0043	1.7362	−0.0336	−0.0038
SD	0.4347	1.7291	0.0413	0.0413	1.6575	0.0415	0.0403	1.6751	0.0419	0.0407
Median	−1.2452	1.8393	−0.0343	−0.0052	1.7552	−0.0320	−0.0039	1.7288	−0.0328	−0.0035
CL90L	−1.9210	−0.1333	−0.0799	−0.0440	−0.1748	−0.0777	−0.0419	−0.2317	−0.0770	−0.0419
CL90U	−0.5695	3.9334	0.0048	0.0318	3.7517	0.0056	0.0329	3.7633	0.0069	0.0345
CL99L	−1.9934	−4.8338	−0.2145	−0.1810	−4.7381	−0.2113	−0.1774	−4.8965	−0.2093	−0.1767
CL99U	−0.5093	8.7393	0.1262	0.1499	8.3735	0.1253	0.1507	8.4597	0.1295	0.1560

In this scenario of monovision, the secondary eye (S) refers to the 'myopic' eye, with a target refraction uniformly distributed in a range from −0.5 to −2.0 dpt, whereas the primary eye (P) was calculated for postoperative emmetropia (TR = 0.0 dpt). In P, the spectacle corrected equals the uncorrected OM (TR = 0.0 dpt); in S, the corrected OM refers to OM with adequate spectacle correction and the uncorrected OM to the OM determined from the chief ray passing through the pupil centre. Mean, SD, Median, CL90L/CL90U, CL99L/CL99U refer to the mean, standard deviation, median and lower/upper boundary of the 90% and 99% confidence interval of the difference S-P.
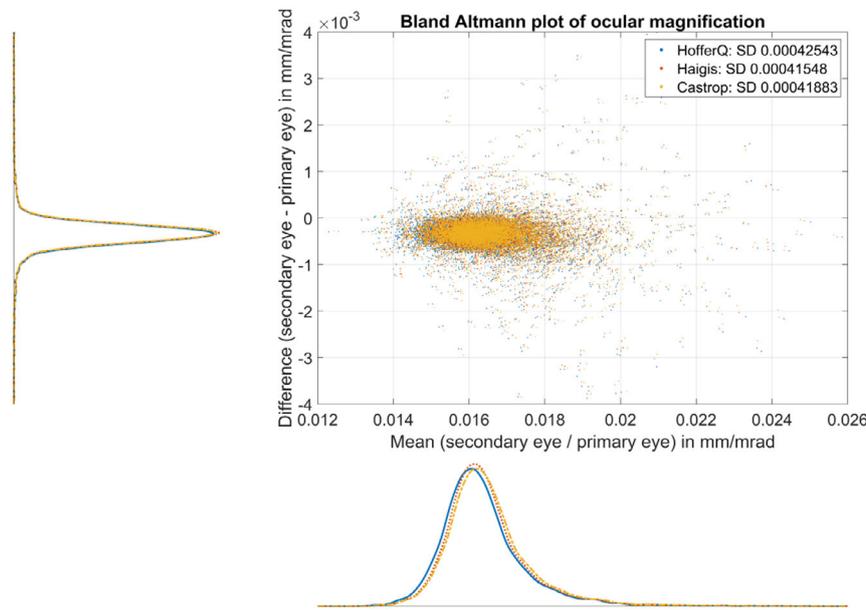

Fig. 3. Scatterhist (combined scatterplot and histogram) of the difference in corrected ocular magnification (S–P) versus the mean of ocular magnification (S/P) for scenario 2 (simulated monovision, P eyes with an IOL power calculated for emmetropia and S eyes with an IOL power for a myopic refraction uniformly distributed between −0.5 and −2.0 dpt. The calculation was performed using 3 pseudophakic models, according to the HofferQ, Haigis, and Castrop formulae. From the histogram on the left of the scatterplot, it can be seen that, as a result of the myopic refraction in S eyes, the ocular magnification in S is systematically smaller compared with P eyes. From the histogram shown below the scatterplot, it can be seen that the Hoffer Q formula (blue line) shows a slight shift to the left indicating that the ocular magnification prediction is slightly lower, whereas the Haigis and the Castrop formulae show slightly larger values for the ocular magnification. The values for the SD of the intra-individual difference (between both eyes of individuals) are included in the legend of the graph.

to the pseudophakic model eye underlying the HofferQ, Haigis and Castrop formulae. The ocular magnifications based on matrix calculation and the prediction using a linear model are plotted on the x and y axes, respectively. Model 1 includes axial length AL, mean corneal power K12 and target refraction TR (according to the HofferQ formula), Model 2 includes AL, anterior chamber depth ACD, mean corneal radius R12 and TR (according to the Haigis formula) and Model 3 includes AL, central corneal thickness CCT, ACD, lens thickness LT, R12 and TR (according to the Castrop formula). The graph shows that prediction of ocular magnification based on a simple linear model is sufficient for clinical routine, and that a matrix calculation is therefore not mandatory.

Discussion

Aniseikonia is well known as a determinant for potential patient dissatisfaction. However, ocular magnification or the disparity of ocular magnification between both eyes (aniseikonia) is widely ignored in modern cataract surgery (Langenbucher & Szentmáry 2008). None of the modern optical biometers on the market (such as the
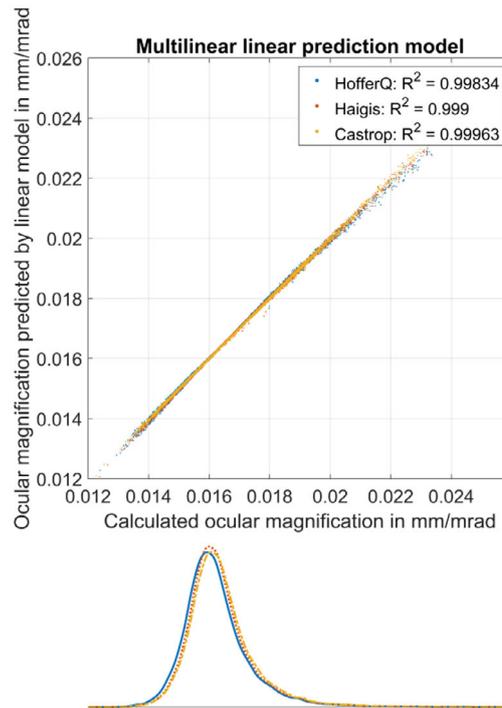
Fig. 4. Scatterhist (combined scatterplot and histogram) for the performance of Model 1, Model 2 and Model 3 according to the pseudophakic model eye underlying the HofferQ, Haigis and Castrop formulae. The ocular magnification based on matrix calculation is shown on the x axis, and the prediction using a linear model is plotted on the y axis. Model 1 includes axial length AL, mean corneal power K12 and target refraction TR, Model 2 includes AL, anterior chamber depth ACD, mean corneal radius R12 and TR and Model 3 includes AL, central corneal thickness CCT, ACD, lens thickness LT, R12 and TR. The input parameters are selected in accordance to the input parameters of the HofferQ, Haigis and Castrop formulae.

IOLMaster 700 or 500, the LenStar, Pentacam AXL or Anterion) can predict post cataract aniseikonia even if the biometric measures captured from both eyes of an individual prior to cataract surgery are available on the machine.

In empirical lens power calculation concepts such as the SRK/SRK2 formulae or artificial intelligence based calculation strategies (*e.g.* the Hill RBF Calculator), the lens power calculation is not based on a pseudophakic model eye. However, in most of the theoretical optical formulae widely used for lens power calculation such as the HofferQ (Hoffer 1980; Hoffer 1993) or Haigis formula (Haigis et al. 2000) there is a direct formulation of the effective axial lens position ELP. In some other classical formulae such as the SRKT or the Holladay formula, there are several correction factors which make a direct derivation of the ELP difficult. However, in all formulae that explicitly estimate the ELP, we can directly obtain a pseudophakic eye model with all the data required for predicting the ocular magnification. If both eyes are measured prior to cataract surgery as generally recommended

to evaluate potential discrepancies in biometric measures (anisometropia), it is a simple task to derive the difference in ocular magnification between both eyes as a measure of aniseikonia (Langenbucher et al. 2003). If restricted to linear Gaussian optics as in most of the lens power calculation concepts, then ocular magnification can be directly extracted from a matrix formulation (Haigis 2009): If we define a refraction matrix for each refracting surface in the pseudophakic eye model and a translation matrix for all interspaces between the refractive surfaces (which are 'filled' with a medium of constant refractive index), then a system matrix calculated from the product of all matrices from right to left (in an inverse order) describes the properties of the optical system when a ray defined by an angle of incidence and offset from the optical axis enters the system. The characteristics of the exiting ray (in terms of angle and offset) are extracted by multiplying the system matrix by the characteristics (angle of incidence and offset) of the entrance ray (Langenbucher et al. 2003). In this context, one element of the system matrix (the lower left element) directly

describes the object to image angular magnification for objects at infinity (defined as the ratio of the tangents of the angles subtended by image and object). In the case of an object at finite distances, the magnification refers to the ratio of image size to object size, and this is described by another element of the system matrix (the lower right element). However, this formulation is restricted to corrected optical systems, where all rays starting from an object intersect at one point in the focal plane. This means that for objects at infinity/at finite distances the lower right/the lower left element of the system matrix equals zero (Langenbucher & Szentmáry 2008). In the general case of an uncorrected optical system (*e.g.* an ametropic pseudophakic eye not corrected by spectacle or contact lenses), both elements in the second row of the system matrix will be nonzero. This means that we have a blurry image in which not all rays passing from the object towards the image plane intersect at a distinct focal point. In this case, we have to select a specific ray for prediction of ocular magnification. In the present study, we have used the chief ray, which passes

through the centre of the pupil. This was assumed to be located at a plane at the front surface of the crystalline lens (at ACD behind the corneal apex) (Langenbucher et al. 2021b).

In the present paper, we used 3 lens power calculation concepts based on linear Gaussian optics (paraxial space) where the ELP and the corneal power can be directly extracted from the formula description. Without loss of generality, formula constants were taken from the IOLCon WEB site (https://iolcon.org) for an aspherical aberration correcting monofocal lens (Hoya Vivinex). In the case of the HofferQ formula (Hoffer 1980; Hoffer 1993), the corneal power is derived from the corneal radius of curvature using the Javal keratometer index, and the ELP is given by 'C + 0.05 mm'. In the Haigis formula (Haigis et al. 2000), the corneal power is calculated from corneal radius using a keratometer index of 1.3315, and the ELP is defined by 'd', which is derived from a linear regression with an intercept (a0), the ACD scaled by a1, and AL scaled by a2. In the modern Castrop formula (Wendelstein et al. 2021; Langenbucher et al. 2021a) first published in 2021, the cornea is considered as a thick lens. In contrast to other concepts, the corneal refractive index of the Liou-Brennan schematic model eye is used to calculate the power of the corneal front and back surface.

In scenario 1, we analysed the biometric parameters from our large dataset. The analysis was restricted to data where measurements of both eyes on the same day were available. From the biometric data, the power of an emmetropic IOL was calculated using the HofferQ, the Haigis and the Castrop Formulae. We then assessed the difference in the biometric measure of both eyes (shown in Table 1) and the intra-individual difference in IOL power and ocular magnification (shown in Table 2). This gives some insight into the intra-individual variation of IOL power and ocular magnification in a 'normal' population after cataract surgery.

In scenario 2, we aimed to simulate the situation of monovision, where in one eye of an individual the IOL implant is calculated for emmetropia and in the other eye the IOL implant is calculated for myopia. In this study—without loss of generality—from the

two eyes of an individual one eye was selected at random as the primary, emmetropic eye and the other as the secondary, myopic eye assuming a uniformly distributed myopia in a range of −0.5 to −2.0 dpt. The respective IOL power and ocular magnification for scenario 2 are shown in Table 2 together with the intra-individual difference. It can be seen that for a mean difference in target refraction (−1.2456 dpt), the (corrected) ocular magnification is reduced by around 3.4 e-4, which equates to around 2% on average (Table 3). Averaging the results of the 3 formulae under test, 3.88%/8.42%/23.34%/ 51.53%/79.73% of cases are predicted to have ocular magnification 5%/4%/ 3%/2%/1% lower in S compared with P, whereas 6.85% of eyes have larger ocular magnification in the myopic S eye compared with P. This suggests that where there is no clear dominance of one eye, our concept could be easily used to decide which eye should be targeted for far or near vision. In contrast, the uncorrected ocular magnification (if myopia of the secondary eye remains uncorrected) is much smaller and ranges between 0.2 and 0.3%.

This matrix-based calculation of ocular magnification can be easily generalized to a situation having spherocylindric surfaces with axes at random (Langenbucher et al. 2005; Langenbucher et al. 2007). In this case, the refraction and translation matrices, and the system matrix describing the entire optical system from the object to the image, are of dimension 4 × 4. The ocular magnification is expressed by a 2 × 2 submatrix of the system matrix, and the meridional magnification with the respective cardinal meridians can be extracted using eigenvalue decomposition (Langenbucher et al. 2005). Such a calculation concept could be used in general to extract the image distortion, for example, in the case of a spherocylindrical cornea corrected with a toric lens implant.

There are some limitations of the present study: (1) our calculations are based on linear Gaussian optics, which is restricted to the paraxial optical space. This implementation does not consider aberrations of the surfaces or large incident ray angles. (2) We have to be aware that the optical transfer of an object to the retinal plane is only

half of the truth. The disparity of retinal image sizes between both eyes is only one determinant for eikonic problems of the patient. In addition to the optical pathway, we should also consider postprocessing of the retinal image in the retina and later on in the visual cortex, as these could also cause eikonic problems and affect the fusion of both images (Achiron et al. 1997). However, in contrast to the optical pathway, investigation of these neural components of vision is much more complex. (3) The intraocular lens is characterized by its refractive power and considered as a thin lens in our model. In addition, in most lens power calculation concepts, the cornea is also considered as a thin lens (although not in the Castrop formula). Both simplifications may cause some inaccuracies in the prediction of ocular magnification.

In conclusion, routine biometry prior to cataract surgery using modern optical biometers generates all of the relevant measures required for prediction of the pseudophakic ocular magnification, in addition to calculating the intraocular lens power to achieve a given postoperative target refraction. As most of the lens power calculation concepts allow direct extraction of the respective underlying pseudophakic eye model, this model can be directly used to estimate the (spectacle) corrected or (blurry) uncorrected ocular magnification and the resulting aniseikonia by comparing the results of both eyes. We used a large dataset from a modern optical biometer and 3 lens power calculation schemes to show the applicability of ocular magnification prediction, provided data on the mismatch of ocular magnification in a normal population after cataract surgery, and simulated the effect of monovision. Finally, we provided a linear prediction model for application to a simple estimation of ocular magnification from biometric data and the target refraction used for lens power calculation.

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