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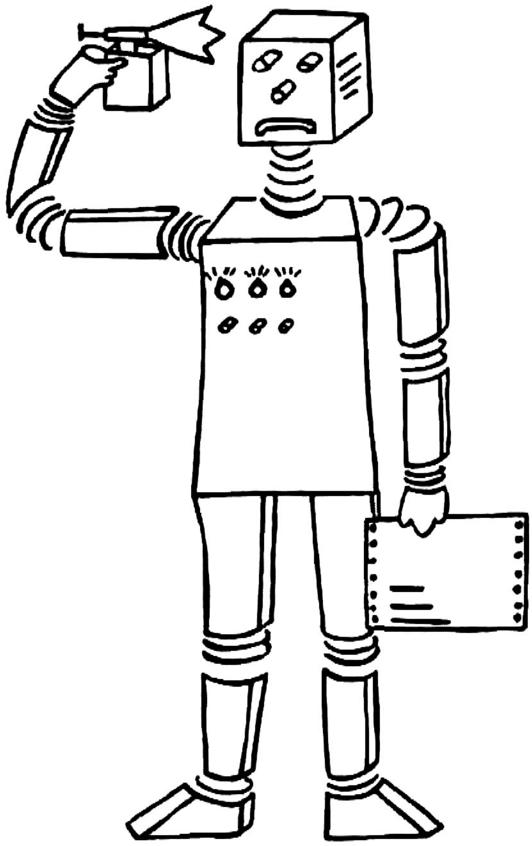

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Abstract

We describe a fragment of Allen's full algebra of time interval relations (the algebra of convex relations) that is useful for describing the dynamic behavior of technical systems. After an intuitive description of the fragment we give two formal definitions and prove that they are equivalent. This provides the basis for the major result of the paper: in a time net in which all interval relations are convex the test for the global consistency of the edge labelling can be carried out in polynomial time (in the general case it is NP-complete). This result makes convex interval relations an attractive candidate whereever qualitative reasoning about technical systems requires testing for global instead of local consistency.

1 Allen's Interval Calculus

Allen's temporal logic [Allen83], [Allen/Hayes85], which was originally developed in order to model temporal references in natural language has since been applied to a multitude of problems, such as planning, discourse representation, and qualitative reasoning.

One of the outstanding features of Allen's logic is the interval calculus which allows partial information about the relative position of time intervals to be represented and propagated. The propagation algorithm in [Allen83] runs in polynomial time (w.r.t. the number of intervals), but it detects only those inconsistencies in the time net which manifest themselves in 3-cliques of intervals whose edges cannot be labelled consistently. [Vilain/Kautz86] have shown that in the general case the test for the global consistency of the edge labelling is NP-complete. At the same time they have demonstrated that in a subalgebra of the full relation algebra 3-consistency implies global consistency, and that therefore in this subalgebra the global consistency check can be carried out in polynomial time, too. Whether the expressiveness of such a subalgebra is sufficient, depends on the task at hand. As an example, the fragment of [Vilain/Kautz86] is insufficient in a planning setting[1].

The aim of this paper is to characterize another subalgebra with the same appealing complexity properties in such a way as to demonstrate its usefulness in representing the dynamic behavior of technical systems.

2 Representing Dynamic Behavior

Representing behavior that varies with time is a ubiquitous task in qualitative reasoning about dynamic systems. Qualitative Physics requires a language in which the envisionment derived from the structural description of a device can be represented. The same language can also be used to store knowledge about the prototypical evolution of faulty behavior in the knowledge base of a diagnostic system. In the literature, several proposals for such a language can be formed, e.g. the graph of process structures in QPT [Forbus84], or the state diagram in ENVISION [de Kleer/Brown84]. Alternatively, one can select a set of

[1] Among others the fragment does not contain {<, m, mi, >} which is essential for the modelling of capacity restrictions.

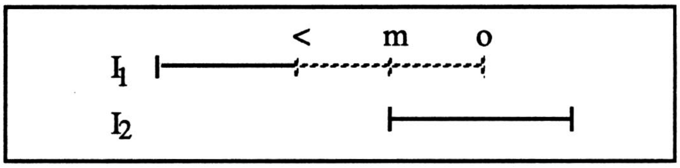
characteristic parameters (temperature, pressure, position etc.) and represent the behavior in the form of one value history[1] for each parameter. Any two consecutive episodes of the same history are related by "meets", whereas the synchronisation between histories is modelled by interval relations between episodes of *different* histories. In an envisionment these "cross relations" mirror the causal dependencies between the components of the system [Voß87]. Such an envisionment corresponds to a real-world behavior only if all the interval relations are globally consistent. In the context of representing dynamic fault situations disjunctions of interval relations can be used to specify the order of episodes insofar as it is characteristic of the situation and to leave room for variation where it is not.[2] Again, the complexity of the global consistency check is of central interest, since it plays a role in event recognition algorithms matching histories against observations.

In both cases histories represent a sequence of events modulo limited imprecision in the relative positions of the intervals. Although in theory any of the 2^{13} disjunctions of primitive interval relations could occur, in practice only relatively few are actually used. Among the disjunctions most frequently found in examples are the following - basically due to their association with causal relationships:

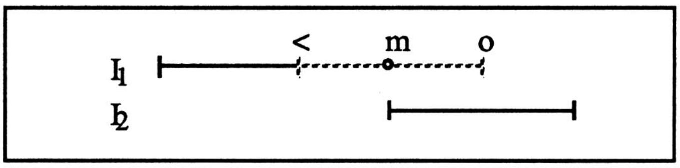
disjunction	verbal interpretation
{ =,s,si,d,oi,f,mi,> }	not-starts-before
{ =,s,si,d,oi,f }	starts-in
{ =,s,si }	starts-simultaneously

All three disjunctions have a convexity property in common: any two t-mappings[3] of a pair of intervals that stand in the relation can be transformed into one another by "continuously deforming" the intervals. In addition, all intermediate stages of the transformation are also t-mappings of the interval.

Consider an example: suppose that the time intervals I_1 and I_2 are specified to stand in one of the relations "before", "meets", and "overlaps". The following diagram shows that the set of positions for I_1's right endpoint in different t-mappings is convex:



Thus, all t-mappings that differ in the position they assign to I_1's right endpoint can be deformed continuously into each other. If we further restrict the relation between I_1 and I_2 to {<, o} we exclude all t-mappings that have I_1's right and I_2's left endpoints coincide. It follows that a t-mapping in which I_1's right endpoint comes before I_2's left endpoint cannot be transformed into one in which it comes after it, because there is an intermediate stage that is not a legal t-mapping.



Such "discontinuous" situations in the dynamic behavior of a technical system - while in principle conceivable - have not been observed in the examples we have studied so far.

Another example for a non-convex disjunction is "not-overlaps" (i.e. {<,m,mi,>}) which plays a role in planning problems and is often cited as an argument for the necessity of keeping the full relation algebra (at least in this context). On the contrary, examples for behaviors of technical devices where the order of two episodes is unspecified but the two may not overlap seem somewhat contrived.

Note finally that the transitive closure of Allen's 13 primitive relations does not contain the convex disjunctions as a subset: both "not-starts-before" and "starts-in" are members of the latter, but not of the former. Neither are convex disjunctions identical to the time point relation algebra of

[1] Histories are sequences of episodes, which are interval-value-pairs.

[2] Stated in another way, disjunctions of interval relations can be viewed as a limited abstraction mechanism for situation descriptions.

[3] Mappings of the intervals' endpoints to a global timeline which conform to the interval relations (a more formal definition is given below).

[Vilain/Kautz86], which contains e.g. the non-convex disjunction {<,o}.

We now investigate the algorithmic status of convex interval relations.

3 Convex Interval Relations

3.1 Formal Definition

In this section we approach the intuitive notion of convexity more formally. We will need some terminology:[1]

DEFINITION: A time net is a pair $<I,C>$, where $I = \{I_1,\ldots,I_2\}$ is a finite set of intervals and $C = \{C_{ij} \mid i,j=1(1)n\}$ is a set of disjunctions of primitive interval relations.

NOTE: The intended semantics of C is C_{ij} being the set of possible relations between intervals I_i and I_j.

DEFINITION: For each interval I let L(I) denote the left endpoint and R(I) denote the right endpoint of I. For every set of intervals I let $P(I) := \{ L(I) \mid I \in I \} \cup \{ R(I) \mid I \in I \}$.

DEFINITION: A t-mapping for a time net $<I,C>$ is a mapping $D: P(I) \rightarrow T$ (T dense, totally ordered, without least or greatest element, e.g. $T = \mathbb{R}$), which respects the relations in C.
Let $D(<I,C>)$ denote the (infinite) set of all t-mappings for $<I,C>$.

We use D(I) as an abbreviation for $<D(L(I)),D(R(I))>$.

DEFINITION: For $p \in P(I)$, $c \in T$ let
$D(<I,C>)\backslash p \mapsto c := \{ D \in D(<I,C>) \mid D(p)=c \}$.

DEFINITION: A set $M \subseteq T \times T$ is interval-convex iff
$(x_1,y_1),(x_2,y_2) \in M \Rightarrow$
$\forall\, x, y \in T$: $[(x_1 \leq x \leq x_2 \vee x_2 \leq x \leq x_1) \wedge (y_1 \leq y \leq y_2 \vee y_2 \leq y \leq y_1) \wedge (x < y) \Rightarrow (x,y) \in M\,]$.

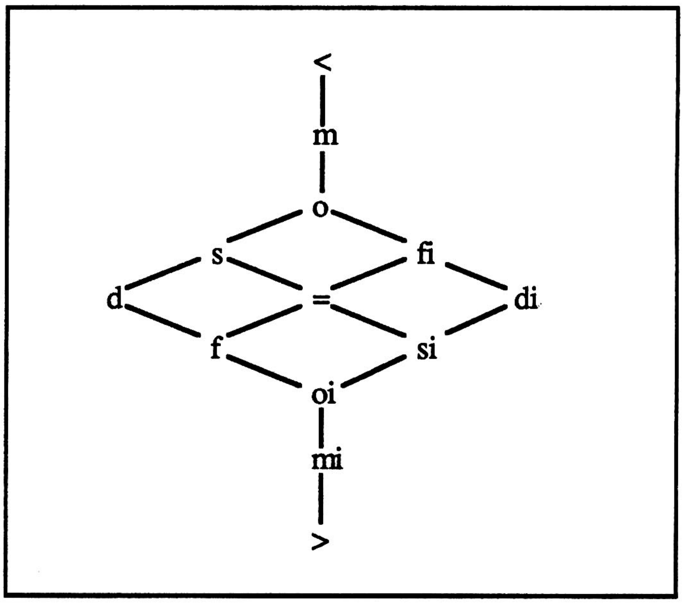
[1] Idenfiers in italics denote sets, all others denote individuals.

Now we can define convex interval relations.

DEFINITION: Let R be a disjunction of primitive interval relations. R is convex, iff $<I=\{I_1,I_2\},C=\{R\}>$ is a timenet and $\exists\, c_1, c_2 \in T, c_1 < c_2$ s.t.
$\{D(I_1) \mid D \in D(<I,C>)\backslash L(I_2) \mapsto c_1 \backslash R(I_2) \mapsto c_2\}$ is interval-convex.

NOTE: There is nothing special about c_1 and c_2. If the condition holds for one pair c_1, c_2 then the same follows for any other pair $c_1 < c_2$. This means that the $\exists$-quantifier could be replaced equivalently by a $\forall$-quantifier. The seeming asymmetry in the definition is resolved in the corollary to theorem 1.

A more graphic account of convex interval relations can be found in the following alternative characterization: in section 2 we appealed to the intuitive notion of transforming the constituent primitive interval relations of a convex disjunction into one another by keeping three of the interval endpoints fixed and continuously shifting the fourth endpoint. We can identify pairs of primitive interval relations which are immediate neighbors in this sense and represent the result in this graph



where relations connected by an edge can be transformed into each other directly (i.e. without going via a third relation). If

we reinterpret the convexity definition in the graph, we get[1]: R is convex iff for any $r_1, r_2 \in R$ all relations along every shortest path from r_1 to r_2 are also members of R.

Alternatively, we can view the graph as the Hasse diagram of an ordering $\sqsubseteq$. Because of the symmetry of the graph, the convex interval relation are the "intervals" w.r.t. $\sqsubseteq$, and we have R is convex iff $\Leftrightarrow \exists\, r_1, r_2$: $R = \{ r \mid r_1 \sqsubseteq r \sqsubseteq r_2 \}$.

By counting these "intervals" we find that there are 82 convex interval relations.

3.2 An Equivalent Representation for Convex Interval Relations

In the subalgebra described in [Vilain/Kautz86] only those relations are admitted which can be represented equivalently in a time net in which the nodes are the time *points* in *P(I)* and the edges are labelled with disjunctions of the primitive time *point* relations <, = and >.

For our purposes we will further exclude {<, >}, the disjunction of the point relations < and >, so that we have PD= $r \in \{<, =, >, \leq, \geq, \text{no-info}\}$ =: *PD* as the set of edge labels in a point-based time net.

DEFINITION: An interval relation R is <u>restricted-endpoint-definable</u> iff:

$$I_1 \, R \, I_2 \Leftrightarrow L(I_1)\, r_1\, L(I_2) \wedge L(I_1)\, r_2\, R(I_2) \wedge R(I_1)\, r_3\, L(I_2) \wedge R(I_1)\, r_4\, R(I_2),\ r_i \in PD\ (i=1(1)4).$$ [2]

Our first result is the following equivalence:

THEOREM 1: Let R be a disjunction of primitive interval relation. R is convex iff R is restricted-endpoint-definable.

[1] Strictly speaking, this claim will be justified only after we have proved theorem 1 below.

[2] The relations $L(I_1) < R(I_1)$ und $L(I_2) < R(I_2)$ are part of the axiomatization of intervals and are therefore not duplicated here.

PROOF: "$\Rightarrow$" Assume that R is convex. We construct an equivalent representation in restricted-endpoint form by choosing $c_1, c_2 \in T$, $c_1 < c_2$, arbitrarily (the construction is independent of the particular choice). From the definitions of "convex" and "interval-convex" it follows that $\{D(I_1) \mid D \in D(\langle I,C\rangle)\backslash L(I_2)\mapsto c_1 \backslash R(I_2)\mapsto c_2\} = (M_1 \times M_2) \cap \{(x,y) \mid x,y \in T, x < y\}$, where M_i are convex subsets of T. Consider M_1 first.

Case I (M_1 singleton):
Necessarily one of $M_1=\{c_1\}$ or $M_1=\{c_2\}$ holds, or else R could not be represented as a disjunction of *interval* relations. If $M_1=\{c_1\}$, add "$L(I_1)=L(I_2) \wedge R(I_1)>L(I_2) \wedge L(I_1)<R(I_2)$" to the restricted-endpoint form, else add "$L(I_1)=R(I_2) \wedge R(I_1)>L(I_2) \wedge R(I_1)>R(I_2)$".

Case II (M_1 non-singleton interval):
Again M_1's endpoints can only be c_1 and c_2, in addition the interval can be unbounded on one or both sides.Each of these cases can be translated into a piece of the restricted-endpoint form (we first deal with the case of an open interval):

$(-\infty,c_1)$	$L(I_1) < L(I_2) \wedge L(I_1) < R(I_2)$
$(-\infty,c_2)$	$L(I_1)$ no-info $L(I_2) \wedge L(I_1) < R(I_2)$
$(-\infty,\infty)$	$L(I_1)$ no-info $L(I_2) \wedge L(I_1)$ no-info $R(I_2)$
(c_1,c_2)	$L(I_1) > L(I_2) \wedge L(I_1) < R(I_2)$
(c_1,∞)	$L(I_1) > L(I_2) \wedge L(I_1)$ no-info $R(I_2)$
(c_2,∞)	$L(I_1) > L(I_2) \wedge L(I_1) > R(I_2)$

If M_1 is closed at one or both of the endpoints, we replace < by ≤ and > by ≥ in the corresponding condition. M_2 is treated analogously.

"$\Leftarrow$" Assume that R is restricted-endpoint definable. Choose $c_1, c_2 \in T$, $c_1 < c_2$, arbitrarily. From the restricted-endpoint form it follows that $I_1\, R\, I_2 \Rightarrow D(L(I_1))\, r_1\, c_1 \wedge D(L(I_1))\, r_2\, c_2$. Let M_1 be the set of $D(L(I_1))$ which satisfy this condition (analogously for $R(I_1)$ and M_2). The sets $\{x \mid x\, r\, c\}$ are convex for arbitrary $r \in PD$, $c \in T$ and this holds for their intersection, too, of course. Combining the results for M_1 and M_2 we get $\{D(I_1) \mid D \in D(\langle I,C\rangle)\backslash L(I_2)\mapsto c_1 \backslash R(I_2)\mapsto c_2\} = (M_1 \times M_2) \cap \{(x,y) \mid x,y \in T, x < y\}$, which implies the conjecture. ■

COLLARY: Using the proof in the $\Leftarrow$-direction it can be shown that $\{D(I_2) \mid D \in D(\langle I,C\rangle)\backslash L(I_1)\mapsto c_1 \backslash R(I_1)\mapsto c_2\}$ is interval-convex if the endpoints of I_1 are kept fixed. Hence,

either of the two dual conditions can be employed in the definition of convex interval relations.

3.3 Complexity of the Global Consistency Check

THEOREM 2: The global consistency check in the algebra of convex interval relations has the same complexity properties as in PREL, the point relation algebra in [Vilain/Kautz86], i.e. it can be carried out in polynomial time.

PROOF: It is sufficient to show that convex interval relations (i) are closed under transitivity and set intersection and (ii) are a subset of PREL. (ii) follows trivially from THEOREM 1 since restricted-endpoint-definable relations are a special case of PREL. (i) can be verified by enumerating all convex relations which is also made possible by THEOREM 1. ■

4 Acknowledgments

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