The Algorithmic Specification Method of Abstract Data Types: An Overview

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1. INTRODUCTION

The practical relevance of abstract data types is twofold. First, their use may support the design of modular programs by stepwise refinement. Second, mechanical correctness proofs of these programs may become feasible in practice.

Classically, a data type is considered to be a multisorted algebra. This algebra consists of carrier sets - one for each sort - and of operations on these sets. Essentially, there exist two different specification methods. An operational specification [Sh 81, Li 81, NY 83] is embedded in an imperative programming language; the carrier sets are built up with the help of the data structures of this programming language and the operations are defined with the help of program pieces. An algebraic specification [GTW 78, GHM 78, BW 82, Re 80, EM 85, etc.] essentially consists of a set of equalities. The algebra thus defined is a special model of this set of equalities, generally the initial model.

A shortcoming of the operational specification method is its lack of abstraction. In fact, an operational specification constitutes an implementation (of a specification) rather than a specification. The main drawback of the algebraic specification method is related to its generality and, in particular, to its non-constructive nature. This leads to the difficult problems of consistency and completeness.

The specification method to be presented here has been called algorithmic to stress its constructive nature. In this method each (new) carrier set is defined by a (constructively defined) formal language; each (new) operation is defined by a recursive program. By its constructive nature the algorithmic specification method essentially avoids the problems of the algebraic specification method. On the other hand it differs from the operational specification method by being more "abstract". Being fundamentally different by their very definition the algebraic and algorithmic specification methods are nevertheless related. For instance, in many "simple" cases algorithmic specifications may be transformed into
algebraic ones by mere rewriting. Other constructive specification methods may be found in [Ca 80, Kl 84].

In the frame of this paper it is not possible to present a complete description of the algorithmic specification method and of its applications. The interested reader is referred to [Lo 84, Lo 81 a, Lo 81 b, Lo 80, Le 85]. Instead, Section 2 explains the main ideas of the method. Section 3 shortly discusses the logic in which properties of the data types may be expressed and proved. Section 4 provides a glimpse of a specification language based on the algorithmic specification method and of its implementation.

2. THE SPECIFICATION METHOD

2.1 The introduction of new sorts

In order to introduce a new sort it is sufficient to define its carrier set and its operations.

The carrier set is defined as a term language, i.e. as a particular formal language. In the case of Figure 1 the carrier set is the term language generated by the operation symbols characterized by the word constructor. The elements of this carrier set are words such as \( c, \text{App}(c, o), \text{App(App}(c, 4), 2) \), etc.

The definition of the operations is different for the constructors and the other operation symbols. The interpretation of the constructors is the Herbrand interpretation. For instance, the value of the operation \text{App} for the arguments \( c \) and \( o \) is the word \text{App}(c, o). The other operation symbols are defined as recursive programs (in the sense of [Ma 74, LS 84]), for instance, \text{Insert} appends an integer to a sequence provided the integer does not yet occur in this sequence; \text{Delete} deletes the rightmost occurrence of an integer; \text{MemberOf} tests whether a (given) integer occurs in a sequence.

2.2 The operations subset and quotient

The method of Section 2.1 does not allow to specify "elaborate" sorts such as the sort consisting of (finite) sets of integers). In fact, let us try to modify the specification of Figure 1, being understood that a term such as \text{App(App}(c, o), 1) now stands for the set \( \{o, 1\} \). Clearly, a term with "duplicates" such as

\[ \text{App(App}(c, o), o) \]
sort seq

signature

  constructor ε : + seq
  constructor App : seq × int + seq
  Insert : seq × int + seq
  Delete : seq × int + seq
  Memberof : seq × int + bool
  Subset : seq × seq + bool

operations

  Insert(s,i) = if Memberof(s,i) then s else App(s,i) fi
  Delete(s,i) = case s of ε : s
                 App(s',i') : if i=i' then s' else App(Delete(s',i),i') fi esac
  Memberof(s,i) = case s of ε : false
                  App(s',i') : if i=i' then true else Memberof(s',i) fi esac
  Subset(s_1,s_2) = case s_1 of ε : true
                     App(s_1,i) : if Memberof(s_2,i) then Subset(s_1,s_2)
                                   else false fi esac

FIGURE 1: A specification of the sort seq, which consists of finite sequences of integers

may not occur in the carrier set. Moreover, terms such as

  App(App(ε,0),1)

and

  App(App(ε,1),0)

have to stand for the same set. The first of these difficulties is solved by introducing an operation which defines a subset of the carrier set; an example of such an operation is the operation Nodup of Figure 2. The second difficulty is solved by introducing an equivalence relation such as Eq of Figure 2; the carrier set then consists of a set of equivalence classes.

It is well-known that such a subset operation or quotient operation yields an algebra only if the closure condition and the congruence condition respectively are fulfilled (see e.g. [GM 83]). Informally, the closure condition expresses that arguments from the subset lead to a value from the subset; the congruence condition expresses that equi-
signature

Nodup : seq → bool
Eq : seq × seq → bool

operations

Nodup(s) ← case s of
  ε : true
  App(s',i) : if Memberof(s',i) then false
               else Nodup(s') fi esac

Eq(s₁,s₂) = if Subset(s₁,s₂) then Subset(s₂,s₁)
            else false fi

FIGURE 2: The specification of Figure 1 together with Nodup as subset operation and Eq as quotient operation defines the sort consisting of finite sets of integers. The operation App is not put at the disposal of the user of the specification, as it does not fulfil the closure condition.

Valent arguments lead to equivalent values. These conditions have to be fulfilled by all operations put at the disposal of the user of the specification; they have to be proved once and for all by the designer of the specification.

3. THE LOGIC OF STRICT ALGEBRAS

The logic of strict algebras [Lo 84] allows to express and prove properties of the data types introduced by algorithmic specifications. The logic is similar to LCF (see [Mi 72, LS 84]) and takes account of the fact that the operations constitute partial computable functions. An example of a formula referring to Figure 1 is:

∀s∈ seq, i ∈ int. Memberof(s,i) = false ⇒ Delete(s,i) = s

An important proof rule is structural induction on the carrier set [Lo 80, Lo 84]. For "non-trivial" properties it may be necessary to use fixpoint induction [Lo 81 b].
4. THE SPECIFICATION LANGUAGE OBSCURE

4.1 The language

The specification language OBSCURE [Le 85] is similar to CLEAR [Sa 84] but is based on the algorithmic specification method instead of the algebraic one. Essentially OBSCURE allows to construct an algebra from a given one.

OBSCURE contains among others three constructs which correspond to the extension described in Section 2.1, to the subset operation and to the quotient operation respectively. Moreover OBSCURE allows the specification and use of parameterized data types. Finally, it provides the possibility to define implementations of abstract data types.

4.2 The implementation

An implementation of OBSCURE is under construction at the university of Saarbrücken. It consists of a programming and a verification part.

The programming part supports the top-down development of programs by stepwise refinement. In particular, it generates the closure and congruence conditions which are to be proved.

The verification part allows interactive correctness proofs and bears similarities with the AFFIRM system [Mu 80, Th 79, Lo 80]. Essentially, the computer performs the formula manipulation and takes care of the administration; the user is responsible for the choice of the proof strategy. The readability of the intermediate results produced by the computer is given particular attention to.
REFERENCES


[Mi 72] Milner, R., Implementation and application of Scott's logic for computable functions. Proc. ACM Conf. on Proving Assertions about Programs, SIGPLAN Notices 7 (Jan. 1972), 1 - 6


