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A Hybrid Approach for Modeling Uncertainty in Terminological Logics

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A Hybrid Approach for Modeling Uncertainty in Terminological Logics*

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Abstract

This paper proposes a probabilistic extension of terminological logics. The extension maintains the original performance of drawing inferences in a hierarchy of terminological definitions. It enlarges the range of applicability to real world domains determined not only by definitional but also by uncertain knowledge. First, we introduce the propositionally complete terminological language $\mathcal{ALC}$. On the basis of the language construct "probabilistic implication" it is shown how statistical information on concept dependencies can be represented. To guarantee (terminological and probabilistic) consistency, several requirements have to be met. Moreover, these requirements allow one to infer implicitly existent probabilistic relationships and their quantitative computation. By explicitly introducing restrictions for the ranges derived by instantiating the consistency requirements, *exceptions* can also be handled. In the categorical cases this corresponds to the overriding of properties in nonmonotonic inheritance networks. Consequently, our model applies to domains where both term descriptions and non-categorical relations between term extensions have to be represented.

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Contents

1 Introduction

Research in knowledge representation led to the development of terminological logics [31] which originated mainly in Brachman's KL-ONE [7]. In such languages the terminological formalism (*TBox*) is used to represent a hierarchy of terms (*concepts*) that are partially ordered by a subsumption relation: Concept B is *subsumed by* concept A, if, and only if, the set of B's real world objects is necessarily a subset of A's world objects. In this sense, the semantics of such languages can be based on set theory. Two-place relations (*roles*) are used to describe concepts. In the case of *defined* concepts, restrictions on roles represent both necessary and sufficient conditions. For *primitive* concepts, only necessary conditions are specified. The algorithm called *classifier* inserts new generic concepts at the most specific place in the terminological hierarchy according to the subsumption relation. Work on terminological languages further led to *hybrid* representation systems. Systems like BACK, CLASSIC, LOOM, KANDOR, KL-TWO, KRYPTON, MESON, SB-ONE, and YAK (for an overview and analyses see [23,36]) make use of a separation of terminological and assertional knowledge. The assertional formalism (*ABox*) is used to represent assertions about the real world. The mechanism for finding the most specific generic concept an object is an instance of and to maintain consistency between ABox and TBox is called the *realizer*.

Since, on one hand, the idea of terminological representation is essentially based on the possibility of *defining* concepts (or at least specifying necessary conditions), the classifier can be employed to draw correct inferences. On the other hand, characterizing domain concepts only by definitions can lead to problems, especially in domains where certain important properties cannot be used as part of a concept definition. As argued by Brachman [5] this may happen in "natural" environments (in contrast to "technical/mathematical" environments). The source of this problem is the fact that in natural environments, besides their description, terms can only be characterized as having additional *typical* properties or properties that are, for instance, *usually* true. If such properties are interpreted as being categorical, this can lead to problems concerning *multiple inheritance*. One example that can be used to highlight these problems is known as the "Nixon diamond": quakers are typically pacifist, republicans are typically non-pacifist, and Nixon is known to be both quaker and republican. Modeling these relationships categorically results in the detection of a *contradiction*. However, in the real world such properties often are only *tendencies*, i.e., republicans "usually" are non-pacifist, for example. Tendencies as well as differences in these tendencies cannot be considered in the framework of term definitions. Several attempts have been made to cope with these problems.

Considering "typical" properties led to nonmonotonic inheritance networks, and may be viewed as "cancellation of inheritance links" or "assume to be true unless told otherwise" [45,10,11,30,40]. These approaches work well if exceptions are explicitly known. However, in the case of conflicts the results can be unsatisfactory (i.e., the "multiple extension problem", compare, e.g. [35]).

A solution concerning "usually true" properties is proposed by Shastri [43,44]. He offers a language to represent empirical information about properties of hierarchically ordered concepts. This empirical knowledge is used instead of definitional roles. His system works well in the case of exceptions and for ambiguities. However, the system is built for handling a large amount of statistical data and is not meant to consider terminological and statistical incompleteness. Other related work can be found in [12,13, 26,25,29]. A detailed survey is given by Pearl [34].

In all these proposals, there is no algorithm comparable to the classifier for maintaining the consistency of the terminology and for reorganizing it according to implicitly existing subsumption relationships. It does not exist because concepts cannot be defined by necessary and sufficient conditions. Recent approaches that provide an integration of both term classification and uncertainty representation and that are related to our work are discussed in Section 5.

We propose an extension of terminological logics that allows one to handle the problems discussed above [15,16] and that pursues our earlier investigation [17]. First, we briefly introduce $\mathcal{ALC}$ [41], a propositionally complete terminological language containing the logical connectives conjunction, disjunction and negation, as well as role quantification. By keeping the TBox semantics, which is based on term descriptions, we are able to use the classifier for extending and reorganizing the terminology. In Section 3 we extend $\mathcal{ALC}$ by defining syntax and semantics of *probabilistic implication*, a construct aimed at considering non-terminological knowledge sources and based on a statistical interpretation. As demonstrated in Section 4, on the basis of the terminological and probabilistic knowledge, certain consistency requirements have to be met. Moreover, these requirements allow one to infer implicitly existent probabilistic relationships and their quantitative computation. While this paper mainly focuses on TBox and statistical aspects, the consideration of an ABox would mean the ability to draw inferences about "probabilistic memberships" of instances.

2 The Terminological Language $\mathcal{ALC}$

The basic elements of the terminological language $\mathcal{ALC}$ [41] are concepts and roles (denoting subsets of the domain of interest and binary relations over this domain, respectively). Assume that $\top$ ("top", denoting the entire domain) is a concept symbol, that A denotes a concept symbol, and R denotes a role. Then the concepts (denoted by letters C and D) of the language $\mathcal{ALC}$ are built according to the abstract syntax rule

$$C, D \longrightarrow A \mid \forall R : C \mid \exists R : C \mid C \sqcap D \mid C \sqcup D \mid \neg C$$

To introduce a formal semantics of $\mathcal{ALC}$ we give a translation into set theoretical expressions with $\mathcal{D}$ being the domain of discourse. For that purpose, we define a mapping $\mathcal{E}$ that maps every concept description to a subset of $\mathcal{D}$ and every role to a subset of $\mathcal{D} \times \mathcal{D}$ in the following way:

1. $\mathcal{E}[\![\top]\!] = \mathcal{D}$
2. $\mathcal{E}[\![\forall R : C]\!] = \{x \in \mathcal{D} | \forall (x,y) \in \mathcal{E}[\![R]\!] : y \in \mathcal{E}[\![C]\!]\}$
3. $\mathcal{E}[\![\exists R : C]\!] = \{x \in \mathcal{D} | \exists (x,y) \in \mathcal{E}[\![R]\!] : y \in \mathcal{E}[\![C]\!]\}$
4. $\mathcal{E}[\![C \sqcap D]\!] = \mathcal{E}[\![C]\!] \cap \mathcal{E}[\![D]\!]$
5. $\mathcal{E}[\![C \sqcup D]\!] = \mathcal{E}[\![C]\!] \cup \mathcal{E}[\![D]\!]$
6. $\mathcal{E}[\![\neg C]\!] = \mathcal{D} - \mathcal{E}[\![C]\!]$

Concept descriptions are used to state necessary, or necessary and sufficient conditions by means of specializations "$\sqsubseteq$" or definitions "$\doteq$", respectively. Assuming symbol A and concept description C, then "$A \sqsubseteq C$" means the inequality $\mathcal{E}[\![A]\!] \subseteq \mathcal{E}[\![C]\!]$, and "$A \doteq C$" means the equation $\mathcal{E}[\![A]\!] = \mathcal{E}[\![C]\!]$. A set of well formed concept definitions and specializations forms a *terminology*, if every concept symbol appears at most once on the left hand side and there are no terminological cycles [24]. A concept C_1 is said to be *subsumed by* a concept C_2 in a terminology $\mathcal{T}$, iff the inequality $\mathcal{E}[\![C_1]\!] \subseteq \mathcal{E}[\![C_2]\!]$ holds for all extension functions satisfying the equations introduced in $\mathcal{T}$. $\mathcal{ALC}$ is used in the implemented prototype system $\mathcal{KRIS}$ [1] where all reasoning facilities such as computing the subsumption hierarchy are realized by sound and complete algorithms.

Terminological languages as $\mathcal{ALC}$ can be usefully applied to definitional world knowledge. For instance, we may introduce

Example 1

$$\begin{array}{rcl} \textit{animal} & \sqsubseteq & \top \\ \textit{flying} & \sqsubseteq & \top \\ \textit{antarctic\_animal} & \sqsubseteq & \textit{animal} \\ \textit{bird} & \doteq & \textit{animal} \sqcap (\forall \textit{moves\_by} : \textit{flying}) \\ \textit{antarctic\_bird} & \doteq & \textit{antarctic\_animal} \sqcap \textit{bird} \\ \textit{penguin} & \sqsubseteq & \textit{antarctic\_bird} \end{array}$$

To characterize the expressiveness of terminological languages, we will examine the three different relations imaginable between two concept *extensions*, i.e., (i) inclusion, (ii) disjointness, and (iii) overlapping:

$$\text{(i) } \mathcal{E}[\![C_1]\!] \subseteq \mathcal{E}[\![C_2]\!], \quad \text{(ii) } \mathcal{E}[\![C_1]\!] \cap \mathcal{E}[\![C_2]\!] = \emptyset, \quad \text{(iii) } \mathcal{E}[\![C_1]\!] \cap \mathcal{E}[\![C_2]\!] \neq \emptyset \tag{1}$$

The first case *can* be caused by (terminological) subsumption. To express extensional inclusion (i) *without* a subsumption relation on terms, some hybrid systems introduced non-terminological language constructs such as `implication` [21,28] or `assert-rule` [4]. Disjointness (ii) *can* be a terminological property. This is the case if, for instance, the above language construct "concept negation" as contained in the expression $C_1 \sqsubseteq C$, $C_2 \doteq (C \sqcap \neg C_1)$ is used. To express non-terminological disjointness between concepts, some systems use the language construct `disjoint`.

However, the information given in case (iii) cannot be reasonably[1] used in existing terminological logics. It seems to be more suitable to generally consider the "degree

[1] except in stating that concept $C_1 \sqcap C_2$ is not incoherent, i.e., it has a necessarily non-empty extension

of intersection" between the respective concept's extensions and to characterize it using an appropriate technique. The idea behind this generalization is to use a probabilistic semantics.

3 The Probabilistic Extension

In the following we consider only one representative for equivalent concept expressions (such as A, $A \sqcap \top$, $A \sqcap A$). The algebra based on representatives of equivalence classes and on the logical connectives $\sqcap$, $\sqcup$, and $\neg$ is known as *Lindenbaum algebra* of the set $\mathcal{S}$ of concept symbols. We use the symbols $\mathbf{D}$ for the set of concept descriptions and $\mathbf{D}^-$, $\mathbf{D}^- \subseteq \mathbf{D}$, for the set of *atoms* of the Lindenbaum algebra. For every function $\mathcal{E}$ the set of extensions of the elements in $\mathbf{D}^-$ forms a partition of $\mathcal{D}$. $\mathcal{D}$ is assumed to be finite. As a language construct that takes into account *all* cases (1), we introduce the notion of *conditional probabilistic implication* (p-implication), which is a generalization of the above mentioned implication construct:

Definition 1 *An extension function $\mathcal{E}$ over $\mathbf{D}$ satisfies a p-implication $C_1 \xrightarrow{p} C_2$, written $\models_{\mathcal{E}} C_1 \xrightarrow{p} C_2$, iff*

$$p = \frac{|\mathcal{E}[\![C_1 \sqcap C_2]\!]|}{|\mathcal{E}[\![C_1]\!]|}$$

holds for concepts $C_1, C_2 \in \mathbf{D}$.[2]

Proposition 1 *The real-valued set function $P_{\mathcal{E}} : 2^{\mathbf{D}^-} \to [0,1]$, $P_{\mathcal{E}}(\{C_i\}) = p_i$ is a probability function over $\mathbf{D}^-$, iff $\models_{\mathcal{E}} \top \xrightarrow{p_i} C_i$ for all $C_i \in \mathbf{D}^-$. In particular, the following conditions are satisfied:*

$$\begin{aligned} P_{\mathcal{E}}(\mathbf{D}^-) &= 1 \\ P_{\mathcal{E}}(D_i) &\geq 0 \text{ for all } D_i \subseteq \mathbf{D}^- \\ P_{\mathcal{E}}(D_i \cup D_j) &= P_{\mathcal{E}}(D_i) + P_{\mathcal{E}}(D_j) \text{ if } D_i \cap D_j = \emptyset \end{aligned}$$

For every concept expression $C_i \in \mathbf{D}$ there exists a subset $D_i \subseteq \mathbf{D}^-$ of atoms such that $C_i = \bigsqcup D_i$. In this way $P_{\mathcal{E}}$ can be extended to concept expressions. In particular,

$$\begin{aligned} P_{\mathcal{E}}(\top) &= P_{\mathcal{E}}(\bigsqcup \mathbf{D}^-) = 1 \\ P_{\mathcal{E}}(C_i) &\geq 0 \\ P_{\mathcal{E}}(C_i \sqcup C_j) &= P_{\mathcal{E}}(C_i) + P_{\mathcal{E}}(C_j) \text{ if } C_i \sqcap C_j = \bot \end{aligned}$$

hold.

[2] The definition can be extended in such a way that a possible uncertainty about the exact probability value can be represented by means of a subrange of [0,1] (see, e.g., [19] for a general examination of numerical models for handling uncertainty).

Definition 2 $\mathcal{P}_{\mathcal{E}} = \{P_{\mathcal{E}} | \models_{\mathcal{E}} \{C_i \xrightarrow{p_k} C_j\}\}$ *is called the set of consistent probability functions.*

From the above explanations it is obvious that we use the *relative cardinality* for interpreting the notions of conditional probability $P_{\mathcal{E}}(C_2|C_1)$ and probabilistic implication introduced in Definition 1.

Example 2 *Assume the set* $\{A \sqsubseteq \top, B \sqsubseteq \top, C \doteq A \sqcap B\}$ *of terminological axioms. From* $\mathcal{S} = \{\top, A, B\}$ *we obtain* $\mathbf{D}^- = \{\neg A \sqcap \neg B, \neg A \sqcap B, A \sqcap B, A \sqcap \neg B\}$. *Then,* $\mathcal{E}$ *and* $\mathcal{D}$ *with* $|\mathcal{D}| = 100, |\mathcal{E}[\![A]\!]| = 40, |\mathcal{E}[\![B]\!]| = 20, |\mathcal{E}[\![A \sqcap B]\!]| = 10$ *induce a probability function*

$$\begin{aligned} P_{\mathcal{E}} : \neg A \sqcap \neg B &\mapsto 0.5 \\ \neg A \sqcap B &\mapsto 0.1 \\ A \sqcap B &\mapsto 0.1 \\ A \sqcap \neg B &\mapsto 0.3 \end{aligned}$$

Note that in spite of having the same name "implication", our "conditioning semantics $B|A$" is different from that of "logical implication $\neg A \vee B$". However, it can simply be derived that between both the relationship

$$P_{\mathcal{E}}(\neg A \vee B) = P_{\mathcal{E}}(\neg A) + P_{\mathcal{E}}(B|A) P_{\mathcal{E}}(A)$$

holds. Nevertheless, with respect to the propositional completeness of the terminological language it is obvious that the notion of logical implication is covered also by our approach. Note further that non-emptyness of $\mathcal{P}_{\mathcal{E}}$ implies the consistency of the whole knowledge base.

Example 2 shows that, assuming complete knowledge on domain $\mathcal{D}$ and on the cardinalities involved, a probability function $P_{\mathcal{E}}$ over $\mathbf{D}^-$ is induced by the extension function $\mathcal{E}$. However, it is generally more realistic to assume less complete knowledge and cardinalities that are rather relative. In the following, we will concentrate on how to extend such knowledge and how to guarantee consistency.

For illustrating the meaning of Definition 1, assume that an observer examines the flying ability of a real class of birds. When finishing his study he may have learned that, different from the model of Example 1, relation *moves_by:flying* holds only for a certain percentage of the birds. The notion of p-implication now allows a representation of universal knowledge of statistical kind in a way that maintains the semantics of the roles: the new concept *flying_object* is created with role *moves_by* restricted to range *flying*. The uncertainty is represented by a p-implication stating that "a certain percentage $100 \cdot p_1$ of birds are *flying_objects* that, by definition, all move by flying". The now more detailed view to the example world leads to the following revision of Example 1:

Example 3 *(revises Example 1)*

$$
\begin{array}{rcl}
\mathit{animal} & \sqsubseteq & \top \\
\mathit{flying} & \sqsubseteq & \top \\
\mathit{antarctic\_animal} & \sqsubseteq & \mathit{animal} \\
\mathit{bird} & \sqsubseteq & \mathit{animal} \\
\mathit{antarctic\_bird} & \doteq & \mathit{antarctic\_animal} \sqcap \mathit{bird} \\
\mathit{penguin} & \sqsubseteq & \mathit{antarctic\_bird} \\
\mathit{flying\_object} & \doteq & \forall \mathit{moves\_by} : \mathit{flying} \\
\mathit{bird} & \xrightarrow{p_1} & \mathit{flying\_object}
\end{array}
$$

This demonstrates that set theory is sufficient for a consistent semantic basis on which both terminological and probabilistic language constructs can be interpreted. On this basis, the p-implication serves as a generalization of both the "implication" and the "disjoint" construct (now appearing as $A \xrightarrow{1} B$ and $A \xrightarrow{0} B$, respectively) used in many hybrid systems.

Proposition 2 $\forall_{A,B:A\neq\bot,B\neq\bot}$:

$$
\begin{array}{rcll}
B \sqsubseteq A & \Rightarrow & B \xrightarrow{1} A & (2) \\
A \xrightarrow{0} B & \Leftrightarrow & B \xrightarrow{0} A & (3) \\
A \xrightarrow{p} B \text{ and } A \xrightarrow{q} B & \Rightarrow & p = q & (4) \\
A \xrightarrow{p} A & \Rightarrow & p = 1 & (5) \\
\top \xrightarrow{p} A & \Leftrightarrow & P_{\mathcal{E}}(A) = p & (6)
\end{array}
$$

Because of the set theoretic semantics, (2) holds. Consequently, derived extensional relations between concepts are based on the union of both terminological and probabilistic statements. We do not, however, interpret the 1-implication as terminological subsumption. In the case of disjointness the equivalence (3) holds for concepts A, B with non-empty extensions. Following (4), for every pair of concepts there is at most one p-implication. Nontrivial reflexive p-implications do not exist (5). Since domain $\mathcal{D}$ is assumed to be exhaustive with respect to our frame of discernment, $P_{\mathcal{E}}(\top) = 1$ holds. Prior probabilities can be represented by special p-implications (6) pointing along subsumption links from general terms to more special ones.

Note that "pointing from exactly one concept to another one" does not mean a restriction concerning the representation of complex "premises and conclusions of rules":[3] in the propositionally complete language $\mathcal{ALC}$ the domain and range concepts of a p-implication may be constructed by means of the operations negation, conjunction, and disjunction.

[3] This was correctly criticized by Yen and Bonissone [47] when discussing our earlier language presented in [17].

4 Probabilistic Consistency and Inferences

First of all we examine the possible relationships between extensions of simple concepts that are introduced by means of the specialization operator "$\sqsubseteq$". Taking into account concept definitions "$\doteq$" involving the connectives concept negation, conjunction, and disjunction means, however, that some of the results can be strengthened. This is shown in Section 4.2.

4.1 Triangular Cases - Concept Specializations

When representing p-implications, their consistency has to be maintained. The simplest case is that of one isolated p-implication for which the consistency requirements are considered in Proposition 2. The requirements for relative proportions when *three* concepts are involved were examined by Dubois and Prade [9] and Heinsohn and Owsnicki [17]. The *most specific case*, in which non-trivial assertions can be made, is characterized as follows [17]:

Proposition 3 *Assuming concepts A, B, C, subsumption $B \sqsubseteq A$, p-implications $A \xrightarrow{p} C$, $B \xrightarrow{r} C$, and $A \xrightarrow{q} B$, then this knowledge is (statistically) inconsistent, if the three (equivalent) inequalities are violated:*

- *for known r, q:*
$$q \cdot r \leq \; p \; \leq 1 - q \cdot (1-r) \tag{7}$$
- *for known p, q:*
$$\begin{cases} \max(1 - \frac{1-p}{q},\, 0) \leq r \leq \min(\frac{p}{q},\, 1) & \textit{if } q \neq 0 \\ 0 \leq r \leq 1 & \textit{if } q = 0 \end{cases} \tag{8}$$
- *for known p, r:*
$$\begin{cases} 0 \leq q \leq \min(\frac{p}{r}, \frac{1-p}{1-r}) & \textit{if } 0 < r < 1 \\ 0 \leq q \leq 1 - p & \textit{if } r = 0 \\ 0 \leq q \leq p & \textit{if } r = 1 \end{cases} \tag{9}$$

Proof Sketch: From *Law of Total Probability* and equation $P_{\mathcal{E}}(C|A) = P_{\mathcal{E}}(C \sqcap A|A)$ we derive

$$\begin{aligned} P_{\mathcal{E}}(C|A) &= P_{\mathcal{E}}(C|B) \cdot P_{\mathcal{E}}(B|A) + P_{\mathcal{E}}(C|A \sqcap \neg B) \cdot P_{\mathcal{E}}(A \sqcap \neg B|A)) \\ &= P_{\mathcal{E}}(C|B) \cdot P_{\mathcal{E}}(B|A) + P_{\mathcal{E}}(C|A \sqcap \neg B) \cdot (1 - P_{\mathcal{E}}(B|A)) \end{aligned}$$

and from unknown $x \stackrel{\text{def}}{=} P_{\mathcal{E}}(C|A \sqcap \neg B)$ ranging from 0 to 1 the equation

$$p = q \cdot r + x \cdot (1-q), \quad x \in [0,1]$$

from which the equivalent inequalities (7), (8), and (9) can be derived by simple reformulations. Note that due to Definition 1 these equations apply to extensions of concepts. ■

The inequalities specify the range allowed for one probability depending on the other two. To maintain local consistency, their satisfiability has to be proved. If exactly two probabilities are given, the inequalities are applied to derive and to keep the information about the range of the other value. In case (8), the condition $q = 0$ implies the range $r \in [0, 1]$, but states that B has an empty extension.

Example 4 *(Bacchus)*
Assume $P_{\mathcal{E}}(B|\top) = 0.6$ and $P_{\mathcal{E}}(C|B) = 0.8$. So, the knowledge base contains $B \sqsubseteq \top$, $C \sqsubseteq \top$, $\top \xrightarrow{0.6} B$, and $B \xrightarrow{0.8} C$. Application of inequality (7) then leads to $\top \xrightarrow{p} C$ with $p \in [0.48, 0.88]$, i.e., $P_{\mathcal{E}}(C|\top) \in [0.48, 0.88]$.

The quantities employed in Example 4 are exactly those used by Bacchus [3, p. 226]. In his logic LP Bacchus focusses on statistical probabilities but, as Nilsson did in his probabilistic logic [27], uses the "material implication" interpretation "$\neg B \vee C$" for this example knowledge. Because of these different viewpoints, Bacchus' result $[0.4, 0.8]$ is different from that based on "relative cardinalities" and derived in Example 4.

The following theorem examines a more general case of statistical relationships with three concepts involved. This generality results from a substitution of subsumption $B \sqsubseteq A$ used in Proposition 3 by a less categorical p-implication. Concerning the involved set of relative proportions the consistency requirements can be formulated as follows:

Theorem 1 *Assuming concepts A, B, C, p-implications $A \xrightarrow{p} C$, $A \xrightarrow{q} B$, $p, q \neq 0$, $B \xrightarrow{q'} A$, $C \xrightarrow{p'} A$, $C \xrightarrow{r'} B$ and $B \xrightarrow{r} C$, then this knowledge is (statistically) inconsistent, if inequalities*

$$\frac{q'}{q} \cdot \max(0, q + p - 1) \leq r \leq \min(1, 1 - q' + p \cdot \frac{q'}{q}) \tag{10}$$

$$\frac{p'}{p} \cdot \max(0, p + q - 1) \leq r' \leq \min(1, 1 - p' + q \cdot \frac{p'}{p}) \tag{11}$$

are violated.

Proof: The proof (of (10)) is based on the common intersection of the extensions of A, B, and C, for which $\mathcal{E}[\![A \sqcap B \sqcap C]\!] \subseteq \mathcal{E}[\![A \sqcap C]\!]$ and $\mathcal{E}[\![A \sqcap B \sqcap C]\!] \subseteq \mathcal{E}[\![B \sqcap C]\!]$ hold. Since the cardinalities of $\mathcal{E}[\![A \sqcap \neg B \sqcap \neg C]\!]$ and $\mathcal{E}[\![\neg A \sqcap B \sqcap \neg C]\!]$ are completely indeterminate, we derive

$$\begin{aligned}
|\mathcal{E}[\![B \sqcap C]\!]| &\leq |\mathcal{E}[\![B]\!]| - |\mathcal{E}[\![A \sqcap B]\!]| + |\mathcal{E}[\![A \sqcap B \sqcap C]\!]| \\
&\leq |\mathcal{E}[\![B]\!]| - |\mathcal{E}[\![A \sqcap B]\!]| + |\mathcal{E}[\![A \sqcap C]\!]| \\
|\mathcal{E}[\![B \sqcap C]\!]| &\geq |\mathcal{E}[\![A \sqcap B \sqcap C]\!]| \\
&\geq |\mathcal{E}[\![A \sqcap B]\!]| + |\mathcal{E}[\![A \sqcap C]\!]| - |\mathcal{E}[\![A]\!]|
\end{aligned}$$

From Bayes rule it is known that

$$
\begin{aligned}
P_{\mathcal{E}}(B|A) \cdot P_{\mathcal{E}}(A) &= P_{\mathcal{E}}(A|B) \cdot P_{\mathcal{E}}(B) && (12)\\
P_{\mathcal{E}}(C|B) \cdot P_{\mathcal{E}}(B) &= P_{\mathcal{E}}(B|C) \cdot P_{\mathcal{E}}(C) && (13)\\
P_{\mathcal{E}}(C|A) \cdot P_{\mathcal{E}}(A) &= P_{\mathcal{E}}(A|C) \cdot P_{\mathcal{E}}(C) && (14)
\end{aligned}
$$

Taking into account equation (12) the cardinalities can be substituted by p-implications as follows

$$
\begin{aligned}
P_{\mathcal{E}}(B \sqcap C|B) &\leq 1 - P_{\mathcal{E}}(A \sqcap B|B) + P_{\mathcal{E}}(A \sqcap C|A) \cdot \frac{P_{\mathcal{E}}(A)}{P_{\mathcal{E}}(B)}\\
&= 1 - P_{\mathcal{E}}(A|B) + P_{\mathcal{E}}(C|A) \cdot \frac{P_{\mathcal{E}}(A|B)}{P_{\mathcal{E}}(B|A)}\\
&= 1 - q' + p \cdot \frac{q'}{q}\\
P_{\mathcal{E}}(B \sqcap C|B) &\geq P_{\mathcal{E}}(A \sqcap C|A) \cdot \frac{P_{\mathcal{E}}(A)}{P_{\mathcal{E}}(B)} + P_{\mathcal{E}}(A \sqcap B|B) - \frac{P_{\mathcal{E}}(A)}{P_{\mathcal{E}}(B)}\\
&= P_{\mathcal{E}}(C|A) \cdot \frac{P_{\mathcal{E}}(A|B)}{P_{\mathcal{E}}(B|A)} + P_{\mathcal{E}}(A|B) - \frac{P_{\mathcal{E}}(A|B)}{P_{\mathcal{E}}(B|A)}\\
&= p \cdot \frac{q'}{q} + q' - \frac{q'}{q} = \frac{q'}{q} \cdot (p + q - 1)
\end{aligned}
$$

In analogy, (11) can be proved. ∎

Note that, for instance, inequality (8) can be derived from (10) by assuming $q' = 1$. In analogy to the set of equivalent requirements of Proposition 3, e.g., inequality (10) can also be reformulated into equivalent requirements that take into account unknown quantities for p, q, and q'.

The whole set $A \xrightarrow{p} C$, $C \xrightarrow{p'} A$, $A \xrightarrow{q} B$, $B \xrightarrow{q'} A$, $B \xrightarrow{r} C$, and $C \xrightarrow{r'} B$ of p-implications is considered in the following theorem that results directly from Bayes' rule. Note that it does not generalize Theorem 1 but serves as an *additional consistency requirement.* In the case of five consistent p-implications, the (consistent) value of the unknown p-implication is obtained.

Theorem 2 *Assuming concepts A, B, C, p-implications $A \xrightarrow{p} C$, $C \xrightarrow{p'} A$, $A \xrightarrow{q} B$, $B \xrightarrow{q'} A$, $B \xrightarrow{r} C$, and $C \xrightarrow{r'} B$, with $p, p', q, q', r, r' \neq 0$, then this knowledge is (statistically) inconsistent, if equation*

$$1 = \frac{r'}{r} \cdot \frac{p}{p'} \cdot \frac{q'}{q} \tag{15}$$

is violated.

Proof: Using the notation of Theorem 2, equations (12), (13), and (14) are the same as

$$\begin{aligned} q &= q' \cdot \frac{P_{\mathcal{E}}(B)}{P_{\mathcal{E}}(A)} \\ P_{\mathcal{E}}(B) &= P_{\mathcal{E}}(C) \cdot \frac{r'}{r} \\ P_{\mathcal{E}}(A) &= P_{\mathcal{E}}(C) \cdot \frac{p'}{p} \end{aligned}$$

A substitution of $P_{\mathcal{E}}(A)$ and $P_{\mathcal{E}}(B)$ in the first equation by the right hand sides of the last two equations leads to the desired result. ■

This result also applies to the situation given in Theorem 1 where the consistency of five p-implications has to be tested, or where for four probability values the range for the fifth one has to be derived:

Proposition 4 *Assuming concepts A, B, C, p-implications $A \xrightarrow{p} C$, $C \xrightarrow{p'} A$, $A \xrightarrow{q} B$, $B \xrightarrow{q'} A$, and $B \xrightarrow{r} C$, $p', q \neq 0$, then this knowledge is (statistically) inconsistent, if inequality*

$$\frac{q'}{q} \cdot \max(0, q+p-1) \leq r \leq \min(1, \frac{p}{p'} \cdot \frac{q'}{q}, \frac{p}{p'} \cdot \frac{q'}{q} \cdot (1 - p' + q \cdot \frac{p'}{p})) \qquad (16)$$

is violated.

Proof: Inequality (16) follows directly from substituting the unknown r' in (15) by the respective range (11). ■

It is obvious that (16) does not improve the lower bound of the range given by (10).

Example 5 *(Dubois and Prade)*
Assume the notation of Theorem 2, $q' = 0.9$, $q = 0.25$, $p = 0.9$, and $p' = 0.6$. Application of (10) leads to the ranges $[0.54, 1.00]$ and $[0.10, 0.567]$ for r and r', respectively, which are identical to those derived in [9]. This result however may still lead to inconsistent assumptions for r'. Following requirement (15) we get ratio $\frac{r}{r'} = 5.4$ and the new consistent range $[0.10, 0.185]$ for r'. Moreover, to know the exact value of r (r') means to know the exact value of r' (r).

There are several special cases of Proposition 3, and Theorems 1 and 2 that are of interest since they present well-known probabilistic requirements. The expressions put in parentheses are optional:

Proposition 5 $\forall_{A,B:A\neq\bot,B\neq\bot}$:

$$(B \sqsubseteq A, C \sqsubseteq A,) A \xrightarrow{q} B, A \xrightarrow{p} C, B \xrightarrow{0} C \;\Rightarrow\; p + q \leq 1 \tag{17}$$

$$B \sqsubseteq A, C \sqsubseteq B, A \xrightarrow{q} B, A \xrightarrow{p} C, B \xrightarrow{r} C \;\Rightarrow\; p = r \cdot q,\ p \leq q,\ p \leq r \tag{18}$$

$$B \xrightarrow{1} C\,(\textit{given by } B \sqsubseteq C, \textit{ e.g.}), A \xrightarrow{q} B, A \xrightarrow{p} C \;\Rightarrow\; q \leq p \tag{19}$$

$$B \sqsubseteq C_1, B \sqsubseteq C_2, A \xrightarrow{q} B, A \xrightarrow{p_1} C_1, A \xrightarrow{p_2} C_2 \;\Rightarrow\; q \leq \min(p_1, p_2) \tag{20}$$

$$B_1 \sqsubseteq C, B_2 \sqsubseteq C, A \xrightarrow{q_1} B_1, A \xrightarrow{q_2} B_2, A \xrightarrow{p} C \;\Rightarrow\; p \geq \max(q_1, q_2) \tag{21}$$

Note that requirements (20) and (21) are direct consequences of applying requirement (19) twice and that they are closely related to the connectives concept conjunction and disjunction, respectively. For instance, assuming $A = \top$ and $B \sqsubseteq (C_1 \sqcap C_2)$, (20) leads to the upper limit $P_{\mathcal{E}}(B) \leq \min(P_{\mathcal{E}}(C_1), P_{\mathcal{E}}(C_2))$.

By explicitly introducing restrictions for the ranges derived by instantiating the consistency requirements, *exceptions* can also be handled. For illustration, assume in Example 3 the situation in which the p-implication *antarctic_bird* $\xrightarrow{p_3}$ *flying_object* is known. In the absence of further information, all that can be concluded for the "flying proportion of *penguins*" is the range $[0, 1]$. If a derived range is considered not to fit the subconcept, the range can be restricted further. For example, "no penguins fly" is represented by the p-implication *penguin* $\xrightarrow{0}$ *flying_object*, which satisfies the $[0, 1]$-implication obtained from consistency tests. In the categorical cases this corresponds to the overriding of properties in nonmonotonic inheritance networks.

Example 6 *(extending Example 3)*

$$\begin{aligned} \textit{bird} &\xrightarrow{p_1} \textit{flying\_object} \\ \textit{antarctic\_animal} &\xrightarrow{p_2} \textit{flying\_object} \\ \textit{antarctic\_bird} &\xrightarrow{p_3} \textit{flying\_object} \\ \textit{penguin} &\xrightarrow{0} \textit{flying\_object} \end{aligned}$$

Note that some approaches for handling uncertainty in term hierarchies propose principles such as *probabilistic inheritance* operating in analogy to terminological inheritance. Mapped into our framework, the simple case of *direct probabilistic inheritance* would be formulated as follows: "*Given concepts A, B, C, subsumption $B \sqsubseteq A$, p-implication $A \xrightarrow{p} C$ (and no more information), then assume $B \xrightarrow{r} C$ with $r := p$.*" However, for this situation where the extension of B is contained in that of A and a certain proportion of the extension of A is contained in that of C, the only *necessarily true* conclusion that can be drawn is $r \in [0, 1]$, i.e., r remains completely indeterminate. This was clearly pointed out by Dubois and Prade [9].[4] Since in our model the statistical knowledge base covers necessarily true relationships only, this or similar principles will not be employed. Probabilistic

[4]If, instead, the semantics of a p-implication would be based on "uncertain conjecture" (which means the uncertainty of a proposition that can be represented by a first order logic expression and whose truth or falsity cannot be established using the available knowledge), the necessarily true conclusion could be strengthened. For instance, for objects denoted by x and from knowledge $P(\forall x : B(x) \rightarrow A(x)) = 1$ and $P(\forall x : A(x) \rightarrow C(x)) \geq p$ one concludes $P(\forall x : B(x) \rightarrow C(x)) \geq p$ [9].

inheritance aspects, however, play a role in the framework of assertional reasoning where p-implications are used to infer knowledge about *instances of concepts* (see, e.g., [20]).

4.2 Triangular Cases - Concept Definitions

The above section copes with concepts introduced by means of simple terminological axioms involving only the specialization operation "$\sqsubseteq$". So, for instance, Proposition 3 is based on the simple assumption $B \sqsubseteq A$. The associated local consistency requirements, however, have to be strengthened if concept definitions introduced by means of the operation "$\doteq$" are involved.

Proposition 6 *(Concept Negation and Conjunction)*

$$A \xrightarrow{p} B \quad \Leftrightarrow \quad A \xrightarrow{1-p} \neg B \tag{22}$$
$$A \xrightarrow{p} \neg A \quad \Rightarrow \quad p = 0 \tag{23}$$
$$A \xrightarrow{p} C \quad \Leftrightarrow \quad A \xrightarrow{p} A \sqcap C \tag{24}$$

Note that if concept negation is involved, the result $p + q \leq 1$ derived in (17) has to be substituted by $p + q = 1$.

If we consider the situation of a concept conjunction (such as $B \doteq (A \sqcap C)$) in the framework of Proposition 3, inequality (9), e.g., has to be based on the assumption $r = 1$ derived from subsumption $(A \sqcap C) \sqsubseteq C$. This however leads to the p-implication $A \xrightarrow{q} (A \sqcap C)$ with $q \leq p$, a result which is less crisp than that of equivalence (24). The reason for now having the precise result is that specialization $B \sqsubseteq A$ (and p-implication $B \xrightarrow{1} C$) is substituted by concept definition $B \doteq (A \sqcap C)$.

Proposition 7 *The following results are obtained from local (triangular) computations:*

$$P_\mathcal{E}(C|A) \quad = \quad \frac{P_\mathcal{E}(A \sqcap C)}{P_\mathcal{E}(A)} \quad \textit{(Bayes rule)} \tag{25}$$
$$A \xrightarrow{p} (A \sqcap B) \quad \Leftrightarrow \quad A \xrightarrow{1-p} (A \sqcap \neg B) \tag{26}$$
$$B \xrightarrow{0} A: \ (A \sqcup B) \xrightarrow{p} A \quad \Leftrightarrow \quad (A \sqcup B) \xrightarrow{1-p} B \quad \textit{(Concept Disjunction)} \tag{27}$$
$$0 \leq P_\mathcal{E}(A \sqcap C|D) \quad \leq \quad \min(P_\mathcal{E}(A|D), P_\mathcal{E}(C|D)) \tag{28}$$
$$1 \geq P_\mathcal{E}(A \sqcup C|D) \quad \geq \quad \max(P_\mathcal{E}(A|D), P_\mathcal{E}(C|D)) \tag{29}$$

Proof: Equation (25) follows from (24) $P_\mathcal{E}(C|A) = P_\mathcal{E}(A \sqcap C|A)$ and from applying (18) to the specialization hierarchy $A \sqcap C \sqsubseteq A \sqsubseteq \top$. Equivalence (26) is based on

$$A \xrightarrow{p} (A \sqcap B) \overset{(24)}{\Leftrightarrow} A \xrightarrow{p} B \overset{(22)}{\Leftrightarrow} A \xrightarrow{1-p} (\neg B) \overset{(24)}{\Leftrightarrow} A \xrightarrow{1-p} (A \sqcap \neg B)$$

Since $B \xrightarrow{0} A \overset{(24)}{\Leftrightarrow} B \xrightarrow{0} (A \sqcap B) \overset{(26)}{\Leftrightarrow} B \xrightarrow{1} (\neg A \sqcap B)$ $(*)$ holds, equivalence (27) can be derived from

$$\begin{aligned}(A \sqcup B) \xrightarrow{p} A &\overset{(24),(26)}{\Leftrightarrow} (A \sqcup B) \xrightarrow{1-p} (\neg A \sqcap (A \sqcup B)) \\ &\Leftrightarrow (A \sqcup B) \xrightarrow{1-p} (\neg A \sqcap B) \overset{(18),(*)}{\Leftrightarrow} (A \sqcup B) \xrightarrow{1-p} B\end{aligned}$$

Inequalities (28) and (29) follow directly from (20) and (21), respectively. ∎

The following proposition examines the notion of concept conjunction and disjunction in a more complex probabilistic framework. Note that it can be easily generalized to the situation where $\top$ is substituted by an arbitrary concept C:

Proposition 8 *(Additivity)*
(a) From $\top \xrightarrow{q_1} A$, $\top \xrightarrow{q_2} B$, and $\top \xrightarrow{p} (A \sqcap B)$ one obtains on the basis of local (triangular) computations the p-implication $\top \xrightarrow{s} (A \sqcup B)$ with probability range

$$\begin{cases} s = q_1 + q_2 - p & \textit{if } p \textit{ is known} \\ \max(q_1, q_2) \leq s \leq \min(1, q_1 + q_2) & \textit{otherwise} \end{cases} \tag{30}$$

(b) From $\top \xrightarrow{q_1} A$, $\top \xrightarrow{q_2} B$, and $\top \xrightarrow{p} (A \sqcup B)$ one obtains on the basis of local (triangular) computations the p-implication $\top \xrightarrow{s} (A \sqcap B)$ with probability range

$$\begin{cases} s = q_1 + q_2 - p & \textit{if } p \textit{ is known} \\ \max(0, q_1 + q_2 - 1) \leq s \leq \min(q_1, q_2) & \textit{otherwise} \end{cases} \tag{31}$$

Proof: The result (30) for known p is obtained from equations

$$\begin{aligned}
P_\mathcal{E}(A \sqcup B|\top) &\overset{(22)}{=} 1 - P_\mathcal{E}(\neg A \sqcap \neg B|\top) \\
&\overset{(25)}{=} 1 - P_\mathcal{E}(\neg B|\neg A) \cdot P_\mathcal{E}(\neg A|\top) \\
&\overset{(22)}{=} 1 - (1 - P_\mathcal{E}(B|\neg A)) \cdot (1 - P_\mathcal{E}(A|\top)) \\
&\overset{(22),(25)}{=} 1 - (1 - \frac{P_\mathcal{E}(B \sqcap \neg A|\top)}{1 - P_\mathcal{E}(A|\top)}) \cdot (1 - P_\mathcal{E}(A|\top)) \\
&\overset{(25)}{=} 1 - (1 - \frac{P_\mathcal{E}(\neg A|B) \cdot P_\mathcal{E}(B|\top)}{1 - P_\mathcal{E}(A|\top)}) \cdot (1 - P_\mathcal{E}(A|\top)) \\
&\overset{(22)}{=} 1 - (1 - \frac{(1 - \overbrace{P_\mathcal{E}(A|B)}) \cdot P_\mathcal{E}(B|\top)}{1 - P_\mathcal{E}(A|\top)}) \cdot (1 - P_\mathcal{E}(A|\top)) \\
&\overset{(25)}{=} 1 - (1 - \frac{(1 - \overbrace{\frac{P_\mathcal{E}(A \sqcap B|\top)}{P_\mathcal{E}(B|\top)}}) \cdot P_\mathcal{E}(B|\top)}{1 - P_\mathcal{E}(A|\top)}) \cdot (1 - P_\mathcal{E}(A|\top)) \\
&= 1 - (1 - \frac{(1 - \frac{p}{q_2}) \cdot q_2}{1 - q_1}) \cdot (1 - q_1) \; = \; q_1 + q_2 - p
\end{aligned}$$

involving only the local computations (22), (25), and logical reformulations over concept descriptions for which the correct subsumptions are offered by the terminological logic. With assumption $A \xrightarrow{0} B$ (or $B \xrightarrow{0} A$, resp.) the expressions covered by horizontal braces become 0 such that $s = q_1 + q_2$, and for unknown p the right hand side in the otherwise-part is obtained. The left-hand side is given by (29). The proof of result (31) can be obtained analogously.

In the more general case where $\top$ is substituted by an arbitrary concept C, the proof would be based on the concepts $C \sqcap A$, $C \sqcap B$, $C \sqcap A \sqcap B$, and $C \sqcap (A \sqcup B)$ that are all subsumed by C. ■

The main advantage of examining local *triangular cases* as in Theorems 1, 2, and Proposition 6 is that "most" of the inconsistencies are discovered early and can be taken into account just in the *current context* of the three concepts involved. Further, not as yet known p-implications can be generated and the associated probability ranges can be stepwise refined. However, testing local consistency requirements *only* for those concepts that are introduced *explicitly* is no guarantee for *global* probabilistic consistency. How to proceed in such situations is demonstrated in the following example.

Example 7 *Assume the knowledge* $B_i \sqsubseteq A$, $A \xrightarrow{0.5} B_i$, $i = 1,2,3$, $B_1 \xrightarrow{0} B_2$, $B_2 \xrightarrow{0} B_3$, *and* $B_1 \xrightarrow{0} B_3$. *Although global inconsistency is obvious, the four local triangular cases are consistent. Constructing now, e.g., the concept* $B_1 \sqcup B_2$ *for which* $(B_1 \sqcup B_2) \sqsubseteq A$, $A \xrightarrow{1} (B_1 \sqcup B_2)$, *and* $(B_1 \sqcup B_2) \xrightarrow{0} B_3$ *hold, with the help of* $A \xrightarrow{0.5} B_3$ *we get a new local case that is inconsistent.*

In the general case, testing global probabilistic consistency leads to a constraint satisfaction problem on a non-discrete domain (for discrete cases see, e.g., [8,22,38]) and, for every p-implication, to successively computing the intersections of the probability ranges derived on the basis of different local examinations.

Remark 1 *For simplicity, the above examinations have been based on the assumption of knowing exact probability values (such as* p, q, *and* r*). The more general probabilistic framework that checks consistency for ranges can be simply obtained by substituting the variables by ranges (such as* $[p_l, p_u]$, $[q_l, q_u]$, *and* $[r_l, r_u]$*). Consequently, in the involved inequalities for testing local consistency, the lower and upper values have to be used with respect to signs and fractions.*

Beside the studies of Dubois and Prade already mentioned above, work related to our approach of checking probabilistic consistency was presented by Paass [29] for the framework of probabilistic logic (see also Bacchus [3] for a logical formalism dealing with qualitative statistical information). The system INFERNO [37] is based on the idea of bounds propagation. There, the drawn inferences are provably correct but may lead to probability bounds that are too weak [19]. The reader is also referred to [32].

5 Related Work

The importance of providing an integration of both term classification and uncertainty representation[5] was recently emphasized in several publications. However, they differ from each other and also from our proposal. Yen and Bonissone [47] consider this integration from a general point of view which, for instance, does not require a concrete uncertainty model (e.g., probabilistic, fuzzy, Dempster-Shafer [18,19]), while in our approach specific properties of an integration are demonstrated, based on a concrete probabilistic model. In [46] Yen proposes an extension of term subsumption languages to fuzzy logic that aims at representing and handling vague concepts. His approach generalizes a subsumption test algorithm for dealing with the notion of vagueness and imprecision. Since our approach aims at modeling uncertainty, it already differs from Yen's proposal in its general objectives. Saffiotti [39] presents a hybrid framework for representing epistemic uncertainty. His extension allows one to model uncertainty about categorical knowledge, e.g., to express one's belief on quantified statements such as "I am fairly (80%) sure that all birds fly". Note the difference from "I am sure that 80% of birds fly", which is modeled in our present paper and requires a completely different formal basis. The work of Bacchus [2,3] is important because he not only explores the question of how far one can go using *statistical* knowledge but also presents LP, a logical formalism for representing and reasoning with statistical knowledge. In spite of being closely related to our work and being able to represent conditional probabilities, Bacchus does not provide a deep discussion of conditionals and the associated local consistency requirements.

6 Conclusions and Outlook

We have proposed a probabilistic extension of terminological logics that takes into account uncertain knowledge arising when certain properties are, e.g., usually but not categorically true. For this purpose, the notion of *probabilistic implication* based on a statistical interpretation has been introduced. This theoretical approach has several advantages: The construct probabilistic implication opens the way to an integration of strictly terminological knowledge and the possibility of modeling exceptions. These no longer appear as contradictions [5], but as a set of weaker inequalities that guarantees the consistency of probability assignments. Moreover, being based on conditional probabilities, consistency can be checked in the current context of the three concepts involved. By separating terminological and probabilistic knowledge, processes maintaining the consistency of the terminological part remain operational. In fact, probabilistic consistency depends heavily on correct terminological subsumptions as established by the classifier.

Current investigations are related to the further refinement of the rules for testing consistency and to the consideration of assertional (ABox) knowledge. The second aspect

[5] Brachman [6] considers "probability and statistics" as one of the "potential highlights" in knowledge representation.

however has the consequence that two different semantics of probabilities have to be integrated, i.e., we have to cope with both universal (statistical) statements involving probabilities over *domains* and assertions describing particular degrees of belief [14,3]. Furthermore, the way assertions about the real world are taken into account differs from classical hybrid representation systems: even if an instance is known to belong to a concept "with certainty", its belonging to other concepts maybe uncertain. So, as discussed in [17], our framework of terminological and probabilistic knowledge is strongly associated with how to answer questions about *probabilistic memberships* of instances and requires an extension of the "classical" realizer.

The computational costs of the algorithms involved are quite high and need theoretical examination. Confining oneself to those p-implications explicitly introduced is of cubic (worst case) complexity and already allows one to draw non-trivial inferences and to detect local inconsistencies. Global consistency however means taking into account the propositional completeness of the terminological language.

Another investigation concerns the implementation of the presented extension and its integration in, e.g., the prototype system $\mathcal{KRIS}$ [1]. Within the WIP project the extended terminological logic can be applied to model the typical (in the statistical interpretation) behavior of users of technical environments and to quantify preferences in choosing actions and plans.

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