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THRESHOLD STRESSES IN MATERIALS CONTAINING DISPERSED PARTICLES

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Introduction

At low temperatures, a dispersion of particles introduces a threshold stress (or yield strength) for dislocation glide. If the particles, of planar spacing ℓ, are sufficiently strong that they are not cut by the dislocations, the shear stress required to cause flow is known as the Orowan stress, σ_{or}, and is, to a good approximation, given by:

$$\sigma_{or} = \frac{C\,G\,b}{\ell}$$

where G is the shear modulus at the test temperature, b the Burgers vector, and C a constant estimated at 0.84 (Kocks (1)).

At high temperatures there is evidence that the dispersion, if stable, introduces a threshold stress for creep. The detailed study of Shewfelt and Brown (2) for instance, shows that, at a stress of about 0.4 σ_{or}, the creep rate falls steeply with decreasing stress, suggesting a threshold. Numerous other investigators have reported threshold stresses for creep of dispersion-strengthened alloys. Their data are summarised in Table 1.

TABLE 1

Material	Vol-fr. %	σ_{th}/G	Reference	
$Ni-ThO_2$	2	6×10^{-4}	Whittenberger	(11)
$Ni-Cr-ThO_2$	2	7×10^{-4}		
$Ni-Cr-Al-ThO_2$	2	5×10^{-4}		
INCONEL MA-754-Y_2O_3	0.6	$2 \times 10^{-4} \div 8 \times 10^{-4}$		
$Ni-Cr-Al-Y_2O_3$	1	4×10^{-4}	Whittenberger	(12)
INCONEL MA-757-Y_2O_3	0.6	$2 \times 10^{-4} \div 9 \times 10^{-4}$		
$Ni-Cr-ThO_2$	2	9×10^{-4}	Lund and Nix	(10)
$Cu-SiO_2$	2.3 ÷ 9	$1 \times 10^{-4} \div 1.8 \times 10^{-4}$	Shewfelt and Brown	(2)
$Cu-Al_2O_3$	0.5 ÷ 1.5	$1 \times 10^{-5} \div 3.4 \times 10^{-5}$	Burton	(9)
$Au-Al_2O_3$	6.1 ÷ 7.9	$1.3 \times 10^{-5} \div 1.7 \times 10^{-5}$	Sautter and Chen	(8)
Stainless Steel-TiN	5	$6 \times 10^{-4} \div 8 \times 10^{-4}$	Evans and Knowles	(4)

Models for this threshold are complicated by the fact that dislocations, at creep temperatures, can both glide and climb. Shewfelt and Brown (3) analyse the process using a computer simulation in which moving dislocations by-pass a random array of strong spherical obstacles, by

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glide and climb. Their work, which goes far in explaining the observations, is complicated; and subsequent attempts to improve on it (Evans and Knowles (4); Hausselt and Nix (5)) either lose the generality of Shewfelt and Brown's approach, or add to the complexity. In this note, we provide a simple derivation of how the mechanism leads to a threshold and the way in which refinements influence it.

The Origin of the Creep Threshold; The Simplest Picture

We treat the problem using the general ideas of Shewfelt and Brown (3). Consider unit length of dislocation gliding on a slip plane which is studded with obstacles, which we think of as hard particles. At a given shear stress σ_s, the dislocation can bow between, and expand beyond, some pairs of particles (we shall refer to these as "transparent gates"); but at others, more closely spaced, it will stop ("opaque gates"). At the Orowan stress, one third of the gates are transparent (1) and this is sufficient for the dislocation to propagate across the entire plane. The Orowan stress is a measure of the number of particles per unit length of the dislocation, n:

$$n = \frac{1}{\ell} = \frac{\sigma_{or}}{C\,G\,b}$$

Let the stress σ_s be less than σ_{or}. Then creep is possible only if the dislocation climbs over some of the obstacles, allowing it to by-pass the rest. The number it can by-pass is:

$$n^* = \frac{\sigma_s}{C\,G\,b}$$

leaving a fraction

$$\frac{n - n^*}{n} = 1 - \frac{\sigma_s}{\sigma_{or}}$$

to be overcome by climb (number of "climbs necessary").

As a segment of dislocation climbs over a particle, its length increases by $\delta\ell$. Work must be done by σ_s to provide the energy of the extra length. We distinguish two distinct cases: local climb (Fig. 1a), such that the climbing segment has a length which scales as the particle radius r; and general climb (Fig. 1b) such that the climbing segment has a length that scales as ℓ. Equating the work done by σ_s as the dislocation advances* by δx (climbing over n^+ particles as it does so) to the increase in line energy, we obtain:

$$\sigma b\,\delta x = (E_s\,\delta\ell)\,n^+$$

where $E_s \approx Gb^2/2$ is the self energy of the dislocation. Hence, the number of particles which can be overcome by climb at a shear stress σ_s (number of "climbs possible") is:

$$\frac{n^+}{n} = \frac{2C}{\alpha}\frac{\sigma_s}{\sigma_{or}}$$

where the climb resistance $\alpha = \delta\ell/\delta x$ describes the rate of increase of line length as the dislocation climbs over the particle (identical in Shewfelt and Brown).

The lines on Fig. 2 show, at a stress σ_s, the number of climbs necessary for flow and the number of climbs that are possible for different values of α. The lowest stress at which flow is possible (the "threshold stress") is given by their intersection:

*Evans and Knowles (4) replace σ_s by the normal stress σ_n, claiming that it is the more important component of stress. This cannot, in general, be true for local climb since the work done by σ_s scales as ℓ while that done by σ_n scales only as r.

$$\sigma_{th} = \frac{\alpha}{2C + \alpha} \sigma_{or}$$

The climb-resistance, α, is an important parameter. For spherical particles local climb gives a value of up to 2. For $\alpha = 2$, the threshold is $\sigma_{th} = 0.56\ \sigma_{or}$. General climb gives a much lower value (3, 5, 6): it is well approximated by $2r/\ell$, or $\sqrt{6/\pi}\, f^{\frac{1}{2}}$ where f is the volume fraction of particles. The range from $f = 1$ % to $f = 10$ % is shown as a shaded band on Fig. 2; the corresponding threshold is about $0.1\ \sigma_{or}$.

Refinements: Local Climb

Distributed Climb Resistance α

The choice of particles for climb is determined by the random fluctuations in particle spacing, or size, or height of intersection with the slip plane. In this section, these refinements are included, replacing the arbitrary choice of an "average" α for local climb by one based on the properties of a random dispersion of particles.

First, let the glide plane intersect equidistant particles at a random height (Fig. 3a): the height of intersection h/r is uniformly distributed between 0 and 1. The shear stress σ_s creates an angle ϕ between the arms of the dislocation (Fig. 3b) as it leaves the particle, where, by equilibrium:

$$\phi = 2 \cos^{-1} \left(\frac{\sigma_s\, b\ell}{2E_s}\right) = 2 \cos^{-1} \left(\frac{\sigma_s\, C}{\sigma_{or}}\right)$$

Shewfelt and Brown (3) analyse the geometry of local climb at a spherical particle: if the dislocation hits the particle near the poles, the climb resistance α is small; but if it hits near the equator, α is large; for a given stress (and thus value of ϕ) climb is possible only at those particles hit above a certain latitude. Shewfelt and Brown's computations (Fig. 3 in Ref. 3) are very well approximated by the climb condition:

$$\phi < \phi_c = \pi \left(\frac{h}{r}\right)^{\frac{1}{3}}$$

or

$$\frac{h}{r} > \frac{h_c}{r} = \left(\frac{2}{\pi} \cos^{-1} \frac{\sigma_s\, C}{\sigma_{or}}\right)$$

This condition is met at a fraction $(1 - h_c/r)$ of the particles, giving:

$$\frac{n^+}{n} = 1 - \left(\frac{2}{\pi} \cos^{-1} \frac{\sigma_s\, C}{\sigma_{or}}\right)^3$$

Fig. 4 shows this modified criterion for the number of possible climbs (curved continuous line). It intersects the simple criterion for the number of necessary climbs at $\sigma_{th} \approx 0.45\ \sigma_{or}$.

Distributed Climb Resistance α and Link Length ℓ

Dorn et al. (7) give the fraction of gates in a random array of particles, which are penetrable at a normalised stress $s = \sigma_s\, \ell/Gb$ as a function of particle strength ϕ (the breakthrough angle). Their result is:

$$P(\phi, s) = \frac{1}{\pi - \phi} \exp\left[-\left(\frac{1}{2s}\right)^2 (\pi - \phi)\right] \int_0^{\pi-\phi} \exp\left[\frac{1}{2}\left(\frac{1}{2s}\right) (\sin 2\psi - \sin 2(\psi + \phi))\right] d\psi$$

for $\phi \geq \pi/2$

and:

$$P(\phi, s) = \frac{2}{\pi - \phi} \int_0^{\pi/2-\phi} \exp\left[-\left(\frac{1}{2s}\right)^2 \left(\frac{3\pi}{2} - \psi - \tfrac{1}{2}\sin\psi - 2\phi\right)\right] d\psi + \frac{1}{\pi - \phi} \exp\left[-\left(\frac{1}{2s}\right)^2 (\pi - \phi)\right]$$

$$\int_{\pi/2-\phi}^{\pi/2} \exp\left[\tfrac{1}{2}\left(\frac{1}{2s}\right)^2 (\sin 2\psi - \sin 2(\psi + \phi))\right] d\psi$$

for $\psi < \pi/2$

The particle strength ϕ varies with the height of intersection, h. If h is distributed uniformly, then the fraction of links penetrable at stress s is:

$$P(s) = \int_0^1 P\left(\frac{h}{r}(\phi), s\right) d\left(\frac{h}{r}\right)$$

This has been evaluated and is shown as a broken line on Fig. 4. For simplicity, the averaging procedure could have been carried out on ϕ_c giving an average break through angle $\overline{\phi}_c = 3\pi/4$. This corresponds to $\alpha(\overline{\phi}_c) = 2\cos(\overline{\phi}_c/2) = 0.77$. A curve for a constant $\alpha = 0.77$ is shown in the figure. The threshold stress it predicts (intersection with the line for the number of necessary climbs) is only slightly lower than that based on the geometric probability distribution.

Refinements: General Climb

Similar considerations of distributed α and ℓ apply to general climb. If the heights of intersection, h_1 and h_2, of two neighbouring particles are random, then the climb resistance falls to:

$$\alpha = \delta\ell/\delta x = \frac{1}{4r^3} \int_0^{2r}\int_0^{2r} \frac{(h_1 - h_2)^2}{\ell} dh_1\, dh_2 = \frac{2}{3}\frac{r}{\ell}.$$

For volume fractions between 1 and 10 % the threshold will vary from $\sigma_{th} = 0.03\ \sigma_{or}$ to $\sigma_{th} = 0.08\ \sigma_{or}$ (Fig. 4). These thresholds will be even more reduced if the link lengths ℓ are distributed.

Conclusions

We have sought to demonstrate that the creep threshold is sensitive to the assumptions and details of the model used to derive it in one regard only: the degree to which climb is localised at the particles. The results of Shewfelt and Brown (3) are rederived in a simple, approximate way. When climb is local, the threshold stress, for all reasonable sets of assumptions and refinements is:

$$\sigma_{th} \approx 0.3\ \frac{Gb}{\ell}$$

When, instead, climb is general, the threshold falls to:

$$\sigma_{th} \approx 0.04\ \frac{Gb}{\ell}$$

And here, too, the result holds for all reasonable sets of assumptions and refinements. All other refinements alter the results by less than a factor of 2; the scatter in the experiments is usually larger than this.

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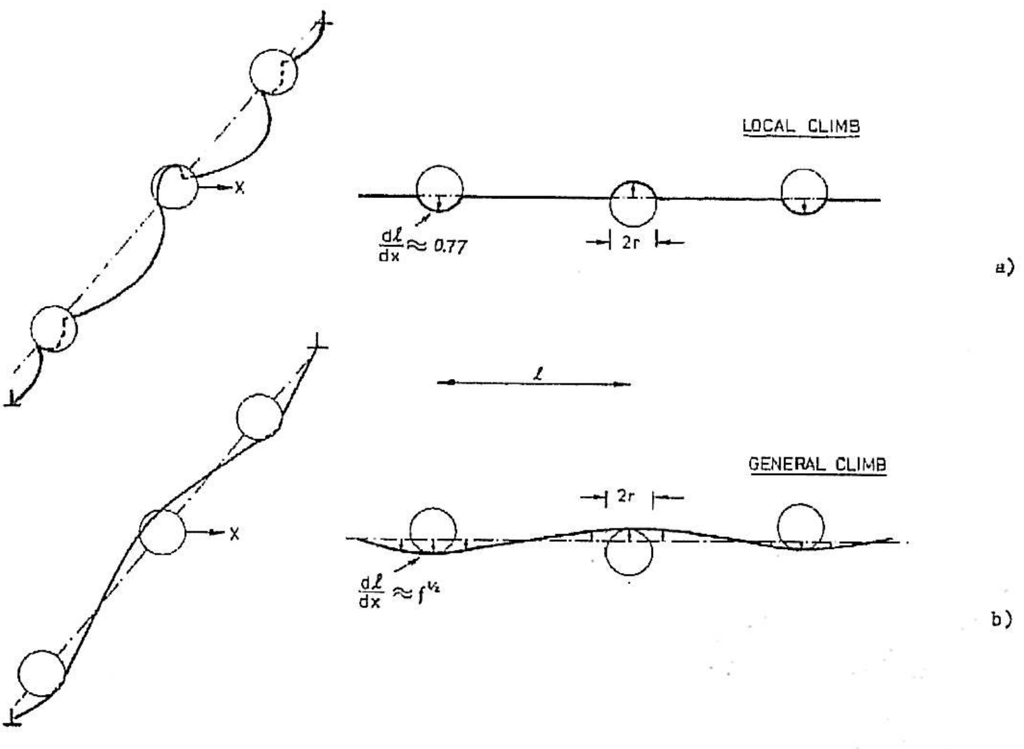


FIG. 1

Distinction between local climb and general climb of a dislocation bypassing particles

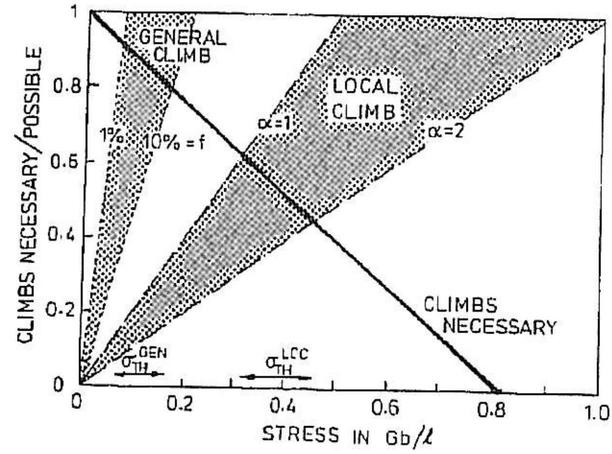
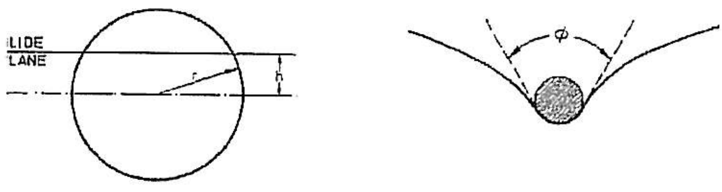


FIG. 2

Fraction of particles to be bypassed by climb (full line - "climbs necessary") and fraction of particles that can be bypassed (for different values of α) at a given shear stress. The intersection determines the threshold stress. f is the volume fraction of particles.



a) b)

FIG. 3

Definition of a) height of intersection h/r of the glide plane with the particle, b) breakthrough angle φ.

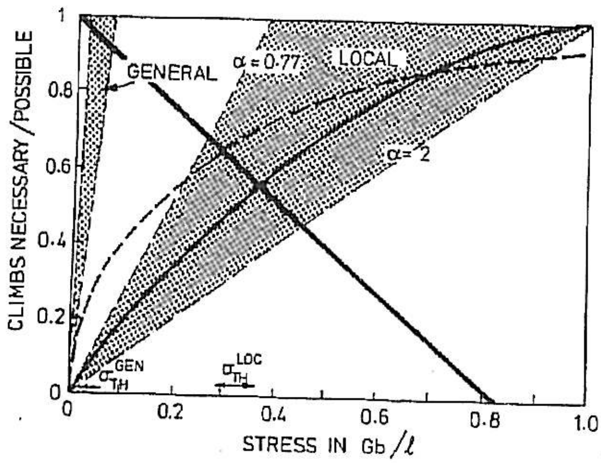


FIG. 4

Same as Fig. 2, with refinements: variable α because of random h height of intersection (full curved line), variable link length (broken line). α = 0.77 is a reasonable average climb resistance. Refinements shift thresholds to lower stresses.